

GEOMETRY AND PHYSICS SEMINAR

Dia 30 Outubro (quarta-feira), às 14h30, sala 6.2.33

On the Asymptotic Distribution of Roots of the Generalised Hermite Polynomials

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Abstract: The generalised Hermite polynomials $H(m,n)$, $m, n \in \mathbb{N}$, form a family of polynomials with applications to random matrices, quantum mechanics and nonlinear wave equations. Central in each of these applications is the fact that these polynomials generate rational solutions of the nonlinear ODE which is the fourth of the celebrated six Painlevé equations. About fifteen years ago, Peter Clarkson published numerical investigations of their roots in the complex plane and posed the problem of describing the asymptotic distribution of the roots as the degree $\deg[H(m,n)] = m \times n$ grows large. In this talk, I will discuss a recent treatment of this problem yielding an explicit, uniform, asymptotic description of the bulk of the roots in terms of elliptic functions. The roots, after proper rescaling, densely fill up a bounded quadrilateral region in the complex plane and, within this region, organise themselves along a deformed rectangular lattice. Our method of attack is a combination of the isomonodromic deformation method and a complex WKB approach to the biconfluent Heun equation.

The seminar is based on two joint papers with Davide Masoero:
Roots of generalised Hermite polynomials when both parameters are large, ArXiv, 2019;
Poles of Painlevé IV Rationals and their Distribution, SIGMA, 2018.

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