Limits of quantum graph operators with shrinking edges

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Abstract: We address the question of convergence of Schrödinger operators on metric graphs with general self-adjoint vertex conditions as lengths of some of graph's edges shrink to zero. We determine the limiting operator and study convergence in a suitable norm resolvent sense. It is noteworthy that, as edge lengths tend to zero, standard Sobolev-type estimates break down, making convergence fail for some graphs. We establish a sufficient condition for convergence which encodes an intricate balance between the topology of the graph and its vertex data. In particular, it does not depend on the potential, on the differences in the rates of convergence of the shrinking edges, or on the lengths of the unaffected edges. In some important special cases this condition is also shown to be necessary.

Before formulating the main results we will review the setting of Schrödinger operators on metric graphs and the characterization of possible self-adjoint conditions, followed by numerous examples where the limiting operator is not obvious or where the convergence fails outright. The talk is based on a joint work with Yuri Latushkin and Selim Sukhtaiev, arXiv:1806.00561 [math.SP].