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Abstract
Current day’s financial crisis and sovereign debt in some countries is imposing new challenges for small and medium sized companies. When focusing on the production sector, the decisions on where to allocate the financial means are becoming increasingly complex and harder to adjust, leading to two main concerns: i) how much and when should we produce; and ii) how should we run the financial commitments in order to succeed with the company and get the best profits. These two parallel concerns usually involve conflicting decisions. In addition, workforce requirements and labor costs play an important role in the interface among the two processes.

In the present paper, we model the two processes (production and cash-flows) in a single framework, using a mixed integer programming discrete-time formulation. When taken individually, each of the problems has been long and deeply discussed in the literature, while the combined version that also incorporates labor financial costs and workforce sizing is scarcer to find.

The main contribution of the paper involves new strategies for financing labor costs, in strong agreement with the company’s production plan and financial commitments. The new strategy intents to relate credit’s ceiling with employment funding, using a sequence of flexible short-term loans.

We consider applications and propose mathematical programming based tools that can be used by companies’ managers for conducting their own solutions analysis, following their own findings and discussion of alternative scenarios.

Keywords: Mixed-integer programming, discrete-time financial planning, production planning, labor financing.

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1. Introduction

Current day’s financial crisis and sovereign debt in some countries is imposing new management challenges for small and medium sized companies. In the particular case involving companies from the production sector, the decisions on where to allocate the financial means are becoming increasingly complex and harder to adjust. In effect, there is a doubt that regularly remains in the manager’s mind – “Have we done the best allocation of financial means between the investment in the production line and the debt service?”. On the other hand, and assuming the same doubt, the bank applies its own procedures to ensure the debt payment, intending to minimize the risk, although expecting that the company manages the business the best way they can. In addition, workforce requirements and costs play an important role in the entire system. While agreeing that unemployment is a strong policy concern in many countries, their costs and legal commitments force most companies to restrict employment increase, which may influence production capacity.

There are a number of cases, in real life, showing that managing production and loans’ duties should run together, with strong commitment with each other. In effect, to succeed with the debt we need a healthy company and, at the same time, the welfare of the company depends on a fruitful relationship with the bank. Kirca & Köksalan (1996) provide a very interesting description of this two-way relationship.

Portugal is one of the countries currently facing a severe sovereign debt crisis as a consequence of the subprime financial crisis, started in the first decade of the current millennium. This work is particularly relevant in the context of small and medium sized companies. In effect, considering the information reported in 2013 by the Instituto Nacional de Estatística\(^2\) (INE), concerning the 2011 economic year, there are 1,135,537 small and medium sized companies (SMCs) in Portugal, over a total of 1,136,697. These companies employed 2,978,383 workers which represent 77\% of the total workforce in the private sector. Yet, the turnover of the SMCs is only 53\% against 47\% disclosed by the larger companies.

Debt crisis in developing countries is a vast theme which requires a very extensive discussion. Yet, instead of conducting a general description over the subject, we concentrate on a very particular aspect that relates labor financing, strategies for employment promotion and healthy management of the tradeoff among production and financial commitments. For

\(^2\) Instituto Nacional de Estatística is the Portuguese Board for Statistics (http://www.ine.pt/xportal/xmain?xpId=INE&xpGId=ine_publicacoes&PUBLICACOESpub_bouI=153408436&PUBLICACOES tema=00&PUBLICACOESmodo=2)
this purpose, we define specific processes for both production and financial streams in a
given time horizon, characterizing a specific framework for the entire process. This
framework embeds the new strategies for labor financing and promotion.

In the present paper, we propose mathematical formulations that attempt to handle both
planning schemes in a single framework, acting together in a discrete-time stream. We
concentrate our objective on the maximization of the profits, respecting a number of
conditions that assure the best financial flow transfer among the two processes: production
line and financial commitments. One of those features involves the determination of the
workforce needs, in line with the best production strategy. The model includes a specific
stream of loans for supporting employment, in order to assist the company’s effort on labor
costs.

When treated separately, discrete-time financial planning and production planning have been
long and deeply studied in the literature. In fact, there is a strong work load on discrete-time
asset/liability management using deterministic, stochastic and multistage stochastic
programming techniques, mainly addressing pension-fund management, banking, portfolio
and insurance industry real-world problems (see, e.g., Hamilton & Moses (1973), Kallberg
et al. (1982), Ashford et al. (1988), Cariño et al. (1994), Klaassen (1998), Cariño & Ziemba
(1998), Cariño et al. (1998), Seshadri et al. (1999), Mulvey & Shetty (2004), Sodhi (2005),
Siegmann & Lucas (2005) and Cornuejols & Tütüncü (2007)). On the other hand, there is
also a vast work on production planning problems and applications, involving capacities on
production, set-up costs, backlogging, lost-sales, multi-item, multi-level and many other
features, exploring a wide diversity of production processes on manufacturing planning and
control systems (see, e.g., Loparic et al. (2001), Pochet (2001), Graves (2002), Pastor et al.
(2009), Absi et al. (2012), Xiao et al. (2012), Toledo et al. (2013)).

Works handling the two problems, incorporating process operations and budgetary
constraints into a supply chain integrated model can be found in Biswas & Narahari (2004),
Guillén et al. (2007), Laínez et al. (2009), Sodhi & Tang (2009), Hahn & Kuhn (2012),
Nickel et al. (2012) and Zevallos et al. (2013), among others. In Guillén et al. (2007), the
authors compare the global model results with those computed by a previous sequential
strategy, in which the operations are first computed and the finances are fitted afterwards.

In Kirca & Köksalan (1996), the authors go even further by incorporating workforce
requirements in a combined financial/production planning model. Their formulation
integrates financial features to support labor costs and production. In addition, the model
addresses a multi-item production process, also involving a multi-level system that
incorporates material purchases to support production. Moreover, they consider hired workers, laid off workers, different types of workers and overtime hours for each class of workers. Other investments, apart from production, are also incorporated in the model. Further works combining financial, production and workforce planning can be found in Fisk (1980) and Muñoz et al. (2000).

The problem discussed in the present paper also incorporates workforce planning in a combined financial/production planning model. In our approach, the production process is not as comprising as the version discussed in Kirca & Köksalan (1996). However, it includes specific loan policies in order to support labor salaries. It involves sequential short-term loans and flexible amortization plans. These short-term contracts should reduce credit risk, while the flexible loans’ payments give the company the ability for better allocating financial means in straight agreement with production and stocking strategies. The production process description lies on a single-item and single-level basis, in order to emphasize the mentioned labor salaries loans scheme.

In the next section, we describe the integrated financial/workforce/production problem under discussion, while proposing mathematical formulations for it in section 3. Case oriented studies and computational tests on the proposed models are discussed in section 4 in which we also conduct various analyses, addressing some case oriented scenarios. The paper ends with a conclusions section.

2. Financial/workforce/production planning problem

The problem under discussion involves a single product or a set of homogeneous products in a production process. We also consider the possibility of hiring new workers, or to plan the company’s workforce needs, regardless of workers’ skills. Two times horizons are set: i) short-term (denoted by $ST$), and ii) long-term (denoted by $LT$), both running over the same stream of fixed time intervals, called time-periods or just periods.

In the short-term horizon, we want to plan production, financial flow and workforce, while the long-term stream represents the expected life-time of the product. The three mentioned concerns are also expected to run over the long-term planning horizon. However, the model here proposed only handles the short-term interval. Then, this time interval can be dragged in order to cover other strips in the long-term horizon, with adequate data updates.
The initial capital comes from the shareholders (\(ic_0\) capital) and from a bank’s loan (\(ln\)). The maturity of this bank’s loan is the long-term horizon (\(LT\)) (say, 60 months, 120 months, etc), and amortizations are made periodically (every \(IT\) period, for \(IT \leq LT\), say every 12 months, for instance) with interest paid in the same occasion. We consider that the amortizations for accomplishing with loan \(ln\) are fixed and the same holds for the interest rate (denoted by \(lr\)). Capital \(ln\) is made available at instant 0, being entirely used in long-term assets, in infrastructures and for initiating the business, while capital \(ic_0\) incorporates capital flows provided by the shareholders, becoming available to start the process.

All the remaining information is defined for the short-term interval, on which the model focuses. There are two streams running in parallel during the short-term scale (from period 1 to \(ST\)): financial planning and production planning. The production requires an adequate workforce load, leading to one of the main concerns in the models: “How many workers should be hired?” This quantity can be adjusted periodically, providing contracts with fixed duration. In this study, we set this time duration to 12 months. Additionally, there are special loan contracts for the company to manage salaries. This aspect represents one of the main novelties of the problems handled in the present paper, and it works this way: the company can borrow a new loan in every period and the amount loaned cannot be greater than the total cost with salaries. Each of these loans has to be fully paid in a very short time interval. For instance, for the loan borrowed in period \(t\), and if the mentioned short-term maturity is 3 months, then the amortizations have to be made in periods \(t+1\), \(t+2\) and \(t+3\) (assuming that \(t+3 \leq ST\), guaranteeing that all salaries’ loans are paid within three months). The amortization process of these loans is made at the company’s desire, which means that the capital can be fully amortized in any of the periods or divided according to the company’s own criteria. In any case, there are interest to pay in each of these periods, over the capital in debt. So, the sooner the loan is paid, the lower are the financial costs. However, the flexibility made available for paying these loans gives the company the ability for allocating the capital at the best benefit of the entire system. In addition, the risk of the bank concerning these loans is reduced due to the very-short time maturities. Furthermore, we may include State aids to the financial cost with these salaries’ loans. In effect, assuming that the banks interest rate for the loan borrowed in period \(t\) is equal to \(sr_1\), and if the State provides a reduction equal to \(sr_2\), then the interest rate to pay is equal to \(sr_t = sr_1 - sr_2\). We also consider that the reduction made available by the State is kept fixed during each 12 months, and then reduced for the very next 12 months interval. This reduction process is
made every 12 months until \( sr_2 = 0 \) (if occurred within the short-time interval under discussion). In what follows, we denote these loans as *salaries’ loans* or *short-term loans*.

So, when looking for the financial stream, and in each period \( t \) of the short-term interval, the in-flow capitals are the cash balance from the previous period (or capital \( iC_0 \) if \( t = 1 \)), the capital loaned in period \( t \) to afford salaries, and the sales net profit in period \( t \). Then, the out-flow capitals are the fixed costs for production; the total costs with salaries; the short-term loans’ amortizations to pay in period \( t \); the interests associated with the loans running in this period; and other cash-flows released outwards in period \( t \), namely dividends paid to shareholders. The remaining amount is the cash balance at the end of period \( t \). In every \( LT \) period (say, every 12 months), we also need to pay loan’s \( ln \) amortizations and interest.

In addition, concerning production, and assuming that we have predicted information about the maximum capability of the market for buying the product, we establish that the amount of product in stock, received from the previous period, plus the amount produced in the current period, should not exceed the maximum demand of the market, while the remaining quantity should flow in stock to the very next period.

To conclude, we should also guarantee that the amount produced in each period \( t \) should not exceed the workforce availability for production, while this workforce amount is bounded by the maximum capacity of production. In addition, there is a minimum number of workers to hire in each 12 months strip.

Figure 1 illustrates some of the various loans to manage in the long-term and in the short-term intervals, previously discussed.

![Diagram](image)

**Figure 1:** The various loans running in the long-term and in the short-term intervals.

In this paper, we discuss the following four versions of the problem previously introduced.

**Problem P1:** we want to maximize the cash balance at the end of the short-term horizon (at the end of period \( ST \)), representing the capital being available after period \( ST \).
**Problem P2:** we want to maximize the sum of all cash balances, in all periods from 1 to $ST$, that is, during the entire short-term interval.

**Problem P3:** we concentrate again in problem’s 1 objective function and assume that the demand is fully accomplished, instead of acting as a maximum expected amount for sales.

**Problem P4:** we consider that the cash-flows released outwards (including dividends) in all periods are no longer known amounts but variables. We also introduce an additional condition, representing sustainability, imposing a lower bound on cash balance at the end of the short-term horizon. This lower bound is set to be at least the current cost with salaries plus the forthcoming amortization and interest payments of the long-term loan (capital $ln$). This time, the objective is to maximize the sum of all cash-flows released outwards, including dividends. Thus, the solution should allocate the capitals the best way, in order to deliver the maximum amount in cash-flows to the outside, while guaranteeing the sustainability condition.

### 3. Financial/workforce/production planning formulations

This section presents mathematical formulations for the problems introduced in Section 2. We start describing the parameters and then we define all variables involved. In order to distinguish parameters from variables, we denote the parameters using two letters plus an index (when appropriate) and variables involve a single letter plus an index (also when appropriate). Parameter’s values are given by the analyst (input values) and variables are the answers that the model is expected to return (output values).

**Index ranges:**

- $LT$: number of periods in the long-term interval
- $ST$: number of periods in the short-term interval
- $IT$: number of periods in the cycle for amortizations and interest payments for the long-term loan. For simplicity of exposition, and with no loss of generality, we assume that $LT$ is a multiple of $IT$.
- $AT$: number of 12 periods sequences for workforce planning ($AT = \lceil LT/12 \rceil$). It represents annual intervals if the periods are assumed to be months.
Parameters:

- $ln$: capital borrowed in the long-term loan
- $ic_0$: initial capital supplied by the shareholders before starting the process
- $dp_t$: maximum expected demand of the product in period $t$, $t = 1, \ldots, ST$
- $cp_t$: maximum capacity of production in period $t$, $t = 1, \ldots, ST$
- $wr_t$: number of units that a worker can produce in period $t$, $t = 1, \ldots, ST$
- $ct_t$: cost per worker in period $t$, $t = 1, \ldots, ST$
- $mw_t$: minimum number of workers in period $t$, $t = 1, \ldots, ST$
- $fc_t$: production fixed costs in period $t$, $t = 1, \ldots, ST$
- $np_t$: sales unitary net profits (excluding fixed costs and labor salaries costs) in period $t$, $t = 1, \ldots, ST$
- $sc_t$: cost for stocking one unit of product in period $t$, $t = 1, \ldots, ST$
- $dv_t$: cash-flows released outwards, including dividends, in period $t$, $t = 1, \ldots, ST$
- $sr_t$: interest rate for the loan started in period $t$, for financing salaries, $t = 1, \ldots, ST$
- $lr$: interest rate of the long-term loan $ln$

Variables:

- $b_t$: cash balance at the end of period $t$, $t = 1, \ldots, ST$
- $y_t$: salaries’ loan borrowed in period $t$, $t = 1, \ldots, ST-1$ (or short-term loans)
- $x_{jt}$: amortization made in period $t$ concerning the salary’s loan started in period $j$,
  \[ j = 1, \ldots, ST-1, \ t = j+1, j+2, j+3, \ t \leq ST \]
- $wk$: number of workers hired in period $k$, $k = 1, \ldots, AT$ (yearly periods)
- $p_t$: number of units produced in period $t$, $t = 1, \ldots, ST$
- $s_t$: stock of product at the end of period $t$, $t = 1, \ldots, ST$

The formulations for problems P2 and P3 involve slight modifications on problem’s P1 model. So, we start describing the formulation for problem P1. In order to help understanding the sequential process involved in both streams: financial and production, we illustrate in Figure 2 all in- and out-flows in each period $t$, for $t = 2, \ldots, ST$, suggesting a 2-level serial system. When $t = 1$, instead of the cash balance at the end of period 0 ($b_0$), we have the initial capital supplied by shareholders before starting the process, denoted by $ic_0$. The solid arrows represent capital flows, while the dashed arrows represent production flows.
Next, we present the formulation for problem P1 (denoted by FP1).

\[
\begin{align*}
\text{maximize} & \quad z = b_{ST} \\
\text{subject to} & \quad s_{t-1} + p_t \leq dp_t + s_t, \quad t = 1, \ldots, ST \\
& \quad p_t \leq wr_t \cdot w_k, \quad t = 1, \ldots, ST, \quad k = \lceil t / 12 \rceil \\
\text{(FP1)} & \quad wr_t \cdot w_k \leq cp_t, \quad t = 1, \ldots, ST, \quad k = \lceil t / 12 \rceil \\
& \quad w_k \geq mw_t, \quad t = 1, \ldots, ST, \quad k = \lceil t / 12 \rceil \\
& \quad y_t \leq ct_t \cdot w_k, \quad t = 1, \ldots, ST-1, \quad k = \lceil t / 12 \rceil \\
& \quad y_t = x_{t+1}, \quad t = 1, \ldots, ST-3 \\
& \quad y_t = x_{t+1}, \quad t = ST-2 \\
& \quad y_t = x_{t+1}, \quad t = ST-1 \\
& \quad ic_0 + y_1 + np_1 \cdot (p_1 - s_1) = fc_1 + dv_1 + ct_1 \cdot w_1 + sc_1 \cdot s_1 + b_1 \\
& \quad b_1 + y_2 + np_2 \cdot (s_1 + p_2 - s_2) = fc_2 + dv_2 + ct_2 \cdot w_1 + sc_2 \cdot s_2 + x_{12} + sr_1 \cdot y_1 + b_2 \\
& \quad b_2 + y_3 + np_3 \cdot (s_2 + p_3 - s_3) = fc_3 + dv_3 + ct_3 \cdot w_1 + sc_3 \cdot s_3 + x_{13} + x_{23} + \\
& \quad + sr_2 \cdot (y_1 - x_{12}) + sr_2 \cdot y_2 + b_3 \\
& \quad b_{t-1} + y_t + np_t \cdot (s_{t-1} + p_t - s_t) = fc_t + dv_t + ct_t \cdot w_k + sc_t \cdot s_t + x_{3,t} + x_{1,t} + x_{2,t} + x_{3,t} + \\
& \quad + sr_{t-1} \cdot (y_{t-3} - x_{t-3,t-2} - x_{t-3,t-1}) + sr_{t-2} \cdot (y_{t-2} - x_{t-2,t-1}) + sr_{t-1} \cdot y_{t-1} + b_t, \\
& \quad t = 4, \ldots, ST, \quad k = \lceil t / 12 \rceil \text{ and } t \mod ST \neq 0
\end{align*}
\]
\[ b_{t-1} + y_t + np_t \cdot (s_{t-1} + p_t - s_t) = fc_t + dv_t + ct_t \cdot w_k + sc_t \cdot s_t + x_{t-3} + x_{t-2} + x_{t-1} + \]
\[ + sr_{t-3} \cdot (y_{t-3} - x_{t-3} - x_{t-3} - x_{t-3} - x_{t-3}) + sr_{t-2} \cdot (y_{t-2} - x_{t-2} - x_{t-2} - x_{t-2}) + sr_{t-1} \cdot y_{t-1} + b_t + \]
\[ + \left( \frac{IT}{LT} \right) \cdot \ln + lr \cdot (1 - \left( \frac{t}{LT} \right)) \cdot \ln , \]
\[ t = 4, \ldots, ST, \quad k = \left\lceil \frac{t}{12} \right\rceil \quad \text{and} \quad t \mod lT = 0 \quad (7e) \]
\[ w_k \in \mathbb{IN}_0, \quad k = 1, \ldots, AT \quad (8) \]
\[ p_t, s_t, b_t, y_t \geq 0, \quad t = 1, \ldots, ST \quad (9) \]
\[ x_{j, t} \geq 0, \quad j = 1, \ldots, ST-1, \quad t = j+1, \ldots, j+3, \quad t \leq ST \quad (10) \]

We assume that \( y_{ST} = 0 \), indicating that there is no borrowed capital in the last period (ST).

From a mathematical standpoint, we could reduce the number of variables and constraints using equalities (6) and (7). However, we chose to keep the model with the proposed representation in order to facilitate its description and discussion.

In model FP1, we want to maximize the profits at the end of the short-term maturity (at the end of period ST), thus, we want to maximize \( z = b_{ST} \). Then, constraints (1) establish that the quantity of product sold in each period \( (s_{t-1} + p_t - s_t) \) does not exceed the associated maximum expected demand \( (dp_t) \), where the quantity sold is equal to the amount produced in the current period \( (p_t) \) plus the stock received from the previous period \( (s_{t-1}) \), minus the stock got left in the current period \( (s_t) \) which becomes available in the very next period.

Constraints (2) are limiting the production concerning workforce availability \( (wr_t \cdot w_k) \), in each period. In addition, this workforce availability is bounded by the maximum capacity of production \( (cp_t) \), being settled in constraints (3). In (4) we impose a lower limit on the number of workers in each period \( (nw_t) \). Then, inequalities (5) state that the capital borrowed for salaries should not exceed the real cost with those payments \( (ct_t \cdot w_k) \), while equalities (6) guarantee the full payment of those loans during the subsequent three months, and within the short-term horizon. The last group of constraints describes cash-flow balance, setting that cash in-flow plus the cash balance received from the previous period should be equal to cash out-flow plus the cash balance attained in the current period. All variables are non-negative, while variables \( w_k \) take integer values, representing the number of workers.

In general terms, the set of inequalities (1) describes the production process, while constraints (3)-(4) characterize workforce limitations. Then, inequalities (2) relate production and workforce variables. In addition, concerning the financial flow description,
equalities (6) are only involved in salaries’ loans, while inequalities (5) relate these loans with the workforce costs. Capital flow conservation is guaranteed in the set of cash-flow balance equalities (7), as mentioned before.

We have assumed that cash balance variables \( b_t \) will only assume non-negative values, forcing the set of solutions to avoid shortfalls. We can argue that this condition is unreal, suggesting that the variables should assume any value in the entire range \( IR \). This aspect will be analyzed in one of the scenarios discussed in Section 4.

In problem P2, we intend to maximize the sum of all cash balances in all periods from 1 to \( ST \), that is, during the entire short-term interval. So, compared with P1, problem P2 involves only a modification on the objective function, while keeping all conditions previously set. Thus, we can model P2 by substituting the previous objective function by the new one

\[
\text{maximize } z = b_1 + b_2 + \cdots + b_{ST},
\]

and keeping all constraints (1)-(10). We denote this model by FP2.

Similarly, we can also formulate problem P3 through a simple modification in FP1. We should recall that P3 requests demand to be fully accomplished, instead of being a maximum quantity for potential selling, as in P1. In this case, we simply need to change constraints (1) to an equality form, becoming

\[
s_{t-1} + p_t = dp_t + s_t, \quad t = 1, \ldots, ST
\]

(1’)

The objective function is the same as in FP1. We can also make a few simplifications in constraints (7), substituting the terms \( s_{t-1} + p_t - s_t \) by the constant \( dp_t \), according to (1’).

Unlike the previous cases, problem P4 involves a few more modifications on model FP1. In this case, we consider that cash-flows released outwards (including dividends) are no longer known amounts but variables. Thus, we ignore parameters \( d_{vt} \) from the problem and introduce the following new set of variables

\[ d_t : \text{cash-flows released outwards (including dividends) in period } t, \quad t = 1, \ldots, ST \]

Furthermore, an additional condition imposing a lower bound on cash balance at the end of the short-term horizon is introduced. This condition states that this lower bound amount is at least the current cost with salaries plus the forthcoming amortization and interest payments of the long-term loan (capital \( ln \)), acting as a sustainability constraint. Then, we want to
maximize the sum of all cash-flows released outwards. This way, the solution should allocate the capitals the best way, returning the maximum amount in cash-flows released outwards (including dividends). Figure 2 can also be used for explaining the production/capital flows involved in this problem, as long as we substitute parameter \( d_v \) by the variable \( d_t \).

Next, we present the formulation for problem P4 (denoted by FP4). In order to avoid repetitions, we use the numbering notation for including the constraints copied from model FP1.

\[
\begin{align*}
\text{maximize} & \quad z = d_1 + d_2 + \cdots + d_{ST} \\
\text{subject to} & \quad (1), (2), (3), (4), (5), (6a), (6b), (6c), (8), (9), (10) \\
& \quad \text{(FP4)} \quad b_1 + y_2 + np_2 \cdot (s_1 + p_2 - s_2) = fc_2 + d_2 + ct_2 \cdot w_2 + sc_2 \cdot s_2 + x_{12} + sr_1 \cdot y_1 + b_2 \\
& \quad b_2 + y_3 + np_3 \cdot (s_2 + p_3 - s_3) = fc_3 + d_3 + ct_3 \cdot w_3 + sc_3 \cdot s_3 + x_{13} + x_{23} + \\
& \quad \quad + sr_1 \cdot (y_1 - x_{12}) + sr_2 \cdot y_2 + b_3 \\
& \quad b_{t-1} + y_t + np_t \cdot (s_{t-1} + p_t - s_t) = fc_t + d_t + ct_t \cdot w_t + sc_t \cdot s_t + x_{t-3,t} + x_{t-2,t} + x_{t-1,t} + \\
& \quad \quad + sr_{t-3} \cdot (y_{t-3} - x_{t-3,t-2} - x_{t-3,t-1}) + sr_{t-2} \cdot (y_{t-2} - x_{t-2,t-1}) + sr_{t-1} \cdot y_{t-1} + b_t , \quad t = 4, \ldots, ST , \quad k = \lceil t/12 \rceil \quad \text{and} \quad t \mod IT \neq 0 \\
& \quad b_{t-1} + y_t + np_t \cdot (s_{t-1} + p_t - s_t) = fc_t + d_t + ct_t \cdot w_t + sc_t \cdot s_t + x_{t-3,t} + x_{t-2,t} + x_{t-1,t} + \\
& \quad \quad + sr_{t-3} \cdot (y_{t-3} - x_{t-3,t-2} - x_{t-3,t-1}) + sr_{t-2} \cdot (y_{t-2} - x_{t-2,t-1}) + sr_{t-1} \cdot y_{t-1} + b_t + \\
& \quad \quad + (IT/LT) \cdot ln + lr \cdot (1 - (t/LT)) \cdot ln , \quad t = 4, \ldots, ST , \quad k = \lceil t/12 \rceil \quad \text{and} \quad t \mod IT = 0 \\
& \quad b_{ST} \geq ct_{ST} \cdot w_{AT} + (IT/LT) \cdot ln + lr \cdot (1 - \lceil ST/IT \rceil) \cdot (IT/LT) \cdot ln \quad (11) \\
& \quad d_t \geq 0 , \quad t = 1, \ldots, ST \quad (12)
\end{align*}
\]

Once again, we assume that \( y_{ST} = 0 \). The new objective function intends to maximize the sum of all cash-flows released outwards. Then, the only changes made in equalities (7) were the substitutions of parameters \( d_v \) by the variables \( d_t \). Also, we have simply included the sustainability constraint (11), stating that the cash balance at the end of the short-term horizon \( b_{ST} \) is greater than or equal to the current cost with salaries plus the forthcoming
amortization and interest payments of the long-term loan (capital $ln$). Conditions (12) impose non-negativity to all $d_i$ variables.

4. Case oriented studies and computational tests

In this section we provide a few practical discussions addressing the problems described in Section 2, using the models proposed in Section 3.

We start considering a general fictitious example, which will be used to analyze the four problems and the associated results. Then, we use this central case for discussing alternative scenarios. As mentioned before, the proposed models are intended to encourage a more regular usage of mathematical programming based tools in order to provide companies’ managers with the possibility of conducting their own solutions analysis and alternative scenarios discussions. The more complex approaches can be complemented in collaboration with trained professionals on mathematical programming, allowing the managers to focus on the companies’ concerns, by analyzing solutions and discussing appropriate scenarios.

For this reason, we discuss the solutions following a decision maker oriented analysis, assuming that the decision maker has some knowledge of the environmental behavior of the problem in hands. In this case, the mathematical model acts as a tool, allowing the decision maker to answer his/her own questions and doubts, by simulating adequate scenarios and analyzing the answers.

An alternative approach could resort to stochastic methodologies. It could be important for discussing the robustness of the solutions, or being used when there are too many uncertain parameters in the problem.

The two approaches do not dominate each other, being complementary, instead. However, in this study, we simply concentrate on the first one, allowing the decision maker to conduct his/her own scenarios, following some of the suggestions in Durbach (2014).

We cannot ignore that the most common solver engine being available in small/medium size companies is the one included in Microsoft/Excel. For this reason, we discuss the models using Excel’s built-in solver. However, if the models become larger, namely on the number of constraints, this solver is no longer capable to answer. To overcome this limitation, we can resort to alternative solver engines for handling larger sized and complex models. Among these alternative choices, there is the lp_solve version v5.5.2.0\(^3\) and the

\(^3\) see, http://web.mit.edu/lpsolve/doc/
ILOG/CPLEX 11.2 package\(^4\). The lp_solve is an open source solver with no limit on model size. It can be used freely, under the GNU lesser general public license conditions. On the other hand, ILOG/CPLEX 11.2 is a commercial product from IBM, being known as a very powerful solver engine. Other third-party powerful solvers that can integrate with Microsoft/Excel include Gurobi, XPress and LINDO, just to name a few.

The experiments were performed under Microsoft Windows 7 operating system on an Intel Core i7-2600 with 3.40 GHz and 8 GB RAM.

All the models discussed in this section were solved using Excel’s build-in solver, except the last tests involving the inflation effect simulation, conducted at the end of Subsection 4.2. The optimums were attained in less than 0.2 seconds, when using Excel’s solver. We have also solved these models using the lp_solve and CPLEX packages. These two solvers took less than 0.025 seconds for running each of the mentioned examples.

### 4.1. Initial problem

The fictitious example used to start discussing the problem involves a 10 years loan \((ln)\) and the financial/production/workforce plan develops on an 18 months stream. Thus, we consider the following time index ranges:

- \(LT = 120\) months (10 years: long-term interval)
- \(ST = 18\) months (short-term interval)
- \(LT = 12\) months (1 year: cycle’s length for long-term amortizations and interest payments)
- \(AT = 2\) years (number of (incomplete) years within the short-term horizon)

In addition, we also consider the following parameters’ values:

- \(ln = 50,000\) € (capital borrowed in the long-term loan)
- \(ic_0 = 10,000\) € (initial capital supplied by the shareholders before starting the process)
- \(dp_t = (1.03)dp_{t-1}\), for \(t = 2, ..., ST\), with \(dp_1 = 1000\) units (the maximum demand of the product starts with 1000 units (in the first period) and grows at a 3\% rate per period)
- \(cp_t = 1500\) units/period (maximum capacity of production in period \(t\), \(t = 1, ..., ST\))
- \(wr_t = 200\) units/worker (number of units that a worker can produce in period \(t\), \(t = 1, ..., ST\))
- \(ct_t = 800\) € (cost per worker in period \(t\), \(t = 1, ..., ST\))
- \(mw_t = 2\) workers/period (minimum number of workers in period \(t\), \(t = 1, ..., ST\))
- \(fc_t = 1500\) €/period (production fixed costs in period \(t\), \(t = 1, ..., ST\))

As mentioned in Section 2, the 

\[ np_t = 8 \, \text{€/unit} \] (sales unitary net profits in period \( t \), \( t = 1, \ldots, ST \))

\[ sc_t = 1 \, \text{€/unit-period} \] (cost for stocking one unit of product in period \( t \), \( t = 1, \ldots, ST \))

\[ dv_t = 2500 \, \text{€/period} \] (cash-flows released outwards in period \( t \), \( t = 1, \ldots, ST \))

\[ sr_t = 0.003 \] (monthly interest rate for the salaries’ loan started in period \( t \), \( t = 1, \ldots, 12 \) (first year))

\[ sr_t = 0.0034 \] (monthly interest rate for the salaries’ loan started in period \( t \), \( t = 13, \ldots, ST \) (second year))

\[ lr = 0.035 \] (yearly interest rate of the long-term loan \( ln \))

As mentioned in Section 2, the \( sr_t \) monthly interest rates for salaries’ loans are defined by \( sr_t = sr_{1t} - sr_{2t} \), where \( sr_{1t} \) represents the bank’s interest rate in period \( t \) and \( sr_{2t} \) represents the State support, being reduced in yearly-time intervals. In this case, we consider that \( sr_{1t} = 0.005 \), while \( sr_{2t} = 0.002 \) during the first year and \( sr_{2t} = 0.0016 \) during the second years.

All solution’s values are rounded to two decimal places.

**Problem P1 solution discussion:**

Problem P1 is intended to maximize the cash balance at the end of the planning horizon (max \( b_{ST} \)). Its optimum solution is given in Table 1.

<table>
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<th>cash-balance (( b_t ))</th>
<th>production (( p_t ))</th>
<th>stock (( s_t ))</th>
<th>salaries’ loans (( y_t ))</th>
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**Table 1:** Problem P1 optimum solution using the initial example parameters’ values.
The maximum cash balance at the end of the 18th month is equal to 9779.86 €. The number of workers to hire is 6 in the first year \((w_1 = 6)\) and 7 during the second year \((w_2 = 7)\).

The solution suggests not using the short-term loans for salaries, when the goal is to maximize the last period cash-balance. It is curious to see this, because it is probably not the most usual decision of a manager having cash availability from banks. However, those loans are not for free, and the solution chooses not to use them if having the chance to manage the problem with the company’s own resources. Yet, we should note that we are not considering opportunity cost into cash, which may lead us to different conclusions, as will be stressed later on.

Graphic 1 shows the cash-balance evolution during the short-term planning interval (variables \(b_t\)). It has a moderate convex variation between periods 1 and 11. Then, it falls severely to 179.86 € and restarts growing at a constant rate of 1. The strong cash-balance decrease in period 12 can be associated to the long-term loan’s amortization and interest payment. It is interesting to observe a cash-balance increase before period 12 in order to support the mentioned payments, in order to avoid shortfalls.

Comparing the cash-balance in month 18 with its homologous (period 6), respectively 9779.86 € and 8435.75 €, we observe a financial growth indicator.

Graphic 2 compares the cash in- and out-flows in each period. These quantities are represented in constraints (7a, 7b, 7c, 7d and 7e), in which the left-hand side comprises the cash in-flow information (excluding the cash-balance variable \(b_{t-1}\)) and the right-hand side involves the cash out-flows (excluding the cash-balance variable \(b_t\)). Actually, after period’s 1 fall down, the cash in-flow grows slightly. Then, in period 12, there is the severe effect of the long-term loan’s payments, and then the two streams keep in balance to the end.
Concerning the production plan, the amounts to produce in each period are bounded by the workforce availability (6 workers $\times$ 200 units = 1200 units in the first year and 7 workers $\times$ 200 units = 1400 units in the second year), which is also bounded by the maximum capacity of production in each period ($cp_t = 1500$ units). This is represented in graphic 3. Between periods 3 and 10, the production exceeds the demand, moving the extra quantities into stock.

In effect, we can also observe that the market demand ($dp_t$ parameters’ values) is not fulfilled after period 10, showing that the best production plan chooses not to produce to the demand boundaries, possibly to avoid extra hiring, as shown in Graphic 4.
Problem P2 solution discussion:
Problem P2 involves the same constraints set as P1, but with a different objective function. In this case, we want to maximize the sum of all period’s cash-balance. Its optimum solution is given in Table 2.

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<th>stock (s_t)</th>
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Table 2: Problem P2 optimum solution using the initial example parameters’ values.

This strategy fits on the managers that usually prefer high levels of capital, even though it may come from borrowed funds. In fact, the optimum solution returns an objective function value (sum of the cash-balances in the entire range) equal to 376,561.43 €. Compared to problem P1 solution, the same function gives 141,813.7 €. In effect, as shown in Graphic 5, the two problem’s cash-balance evolution indicates a stronger load in problem’s P2 solution, with a similar severe fall in period 12. However, in this case, there is a very effective decrease in the last period, in order to pay the large short-term loans still in debt at the end. In fact, problem’s P2 strategy borrows to the limit (number of workers × salary) in all periods, as shown in variables y_t solution values in Table 2, which forces to a very strong financial effort for paying the debts, especially in the last period.
In addition, comparing the last period (month 18) cash-balance in the two solutions, problem P1 brings slightly more money into month 19 than P2, with 9779.86 € and 8845.45 €, respectively. However, if the extra cash is invested in alternative financial products, with a positive return rate, then problem P2 strategy could become profitable, compared to P1. This aspect is not hard to incorporate in the formulations under discussion. Curiously, the two solutions propose hiring the same number of workers.

The solution strategy proposed in P2 is quite common in practice. It brings to the company all available funds. In effect, if a company has the chance of allocating the extra cash into profitable investments, besides those within the business, then P2 strategy can be rewarding. Otherwise, the strategy in P1 should be preferable.

**Problem P3 solution discussion:**

Problem P3 calls again the initial objective function, used in P1 (to maximize the cash-balance in the last period (month 18)). However, in this case, P3 forces the production to fulfill the entire demand, in all periods. Its optimum solution is given in Table 3.

![Graphic 5: Cash-balance optimum values for problems P1 and P2, for the initial example.](image)
The optimum cash-balance in period 18 is equal to 3233.76 €, being much smaller than the same period cash-balance in P1. In fact, the solution in P3 indicates that this strategy is more demanding in a number of aspects, namely in capital, in the number of workers and in stock usage. It forces to hire one more worker, in the first year, than in P1.

Graphic 6 compares the cash-balance in problems’ P1 and P3 solutions, along the entire short-term range, confirming the previous observations. The harder financial conditions for managing problem’s P3 strategy forces the solution to resort to short-term loans for salaries, especially after period 10. In this case, it is interesting to observe the way the solution adjusts those loans and the associated amortization plans. They are called during the harder periods, between months 12 and 17, when cash-balance is null. Note also that the short-term loans are limited to the salaries costs in each period. This limitation was only observed in period 12, being equal to 5600 € (7 workers × 800 €). The entire financial cost with these loans is 125.22 €.
Problem P4 solution discussion:

Problem P4 introduces a new strategy. In this case, the cash-flows released outwards (including dividends) are no longer known amounts but variables. This means that the parameters \( dv_t = 2500 \, \text{€/period} \), used in problems P1 to P3, are substituted by variables \( d_t \), for \( t=1,\ldots,\text{ST} \), letting the optimization process decide on when and how much cash should be released outwards, in each period. Furthermore, and in order to guarantee sustainability, we have introduced an additional condition imposing a lower bound on cash balance at the end of the short-term horizon (constraint (11)). This condition guarantees that the cash entering month 19 (variable \( b_{19} \) value) should be enough for paying one month of salaries and for paying the forthcoming amortization and interest of the long-term loan (capital \( ln \)). Its optimum solution is given in Table 4.

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<th>production ( (p_t) )</th>
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</tr>
<tr>
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<td>125</td>
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<td>0</td>
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<td>0</td>
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<td>0</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>18</td>
<td>12175</td>
<td>0</td>
<td>1400</td>
<td>0</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
</tbody>
</table>

Table 4: Problem P4 optimum solution using the initial example parameters’ values.

The maximum solution value of FP4 is equal to 50,104.86 €, representing the sum of all cash-flows released outwards during the entire short-term range. Compared to the optimum solution of FP1, the same function is equal to 45,000 € \( = \sum_{t=1}^{\text{ST}} dv_t \), indicating that problem
P4 solution releases more money than P1. Still, and due to the flexibility to manage these capitals, last month cash-balance in problem P4 is higher than in P1, being equal to 12,175 € and 9,779.86 €, respectively. This is curious because problem’s P1 objective function is entirely devoted to maximize this quantity, which means that the mentioned flexibility to manage the capitals to release can be most convenient. Graphic 7 compares the cash-balance in problems’ P1 and P4 solutions, along the entire short-term range.

![Graphic 7](image_url)

**Graphic 7:** Cash-balance optimum values for problems P1 and P4, for the initial example.

It is also worth noting that problem P4 solution proposes hiring the same number of workers as P1. Additionally, comparing the entire own capitals, the sum of the cash-balances during the entire short-term range is much smaller in P4 solution than in P1. In this case, instead of keeping unused capitals in the company (applied in profitable investments (as suggested in P2 discussion) or not), the solution chooses to give them to shareholders (for instance), as soon as they become available.

A comparison between the mentioned flexibility for releasing cash-flows and the rigid strategy followed in P1 is depicted in Graphic 8. These cash-flows released include dividends. In fact, problem’s P4 flexible strategy gives the manager the chance to release capitals at the best convenience of the company. As a result, problem’s P4 solution is more generous than problem’s P1.

![Graphic 8](image_url)

**Graphic 8:** Cash-flows released outwards in problems P1 and P4, for the initial example.
4.2. Alternative scenarios discussion

Using the previously discussed initial example, we propose a number of alternative scenarios based on strategically oriented modifications on some selected parameters’ values. In order to avoid an extensive results discussion, we only concentrate on problems P1 and P4, and to a very limited extent. In effect, and as mentioned before, we want to encourage a more regular usage of mathematical programming based tools in order to provide companies’ managers with the possibility of conducting their own solutions analysis and alternative scenarios discussions.

Problem P1 alternative scenarios discussion:

Concerning problem P1, we start discussing alternative values for the initial capital supplied by the shareholders, parameter $i c_0$. In this case, we propose the following values: $i c_0 = 100; 1,000; 2,500; 5,000$ and $20,000$. We assumed that $i c_0 = 10,000$ in the initial example. Graphic 9 compares the cash-balance values of the various solutions’ scenarios, along the entire short-term range. Table 5 reports the objective function values ($\max b_{st}$), the number of workers to hire and the periods when short-term loans are borrowed (signaled in gray).

![Graphic 9: Cash-balance optimum values for problems’ P1 scenarios, involving variations in parameter $i c_0$.](image)

<table>
<thead>
<tr>
<th>$i c_0$ values</th>
<th>max $b_{st}$</th>
<th>num. of workers</th>
<th>short-term loans: starting periods</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1st year</td>
<td>2nd year</td>
</tr>
<tr>
<td>100</td>
<td>impossible</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,000</td>
<td>666.31</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>2,500</td>
<td>2,208.41</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>5,000</td>
<td>4,750.53</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>10,000</td>
<td>9,779.86</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>20,000</td>
<td>19,779.86</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 5: Problem P1 alternative scenarios, involving variations in parameter $i c_0$. 
As expected, when the initial capital supplied by the shareholders decreases, it becomes harder to manage the company’s cash-flows. In effect, when this initial capital is equal to 5,000 € or below, the company was forced to borrow short-term loans, and those loans tend to be essential when cash-flow availability becomes narrower. There are a number of causes for such narrower conditions. One of those is the lack of financial support at the earlier stages of the process, as shown in these scenarios. It is also interesting to observe that the hiring strategy had no changes, among these experiments.

An alternative discussion may involve a comparison between the chance of using the flexible amortizations schemes and the classical forms of loans’ payments. For this purpose, we choose the previously discussed scenario with $i_c = 1,000$, in which short-term loans were strongly borrowed. In addition, and in order to simulate an usual plan proposed by the banks, we force these loans to be paid in equal amounts during the short-term maturity, meaning that $x_{t+j} = y_t / 3$ for $j = 1, \ldots, 3$ and $t = 1, \ldots, ST-3$; $x_{t+j} = y_t / 2$ for $j = 1, \ldots, 2$ and $t = ST-2$; and $x_{t+1} = y_t$ for $t = ST-1$. Thus, the optimum solution of P1 with these amortizations plan and $i_c = 1,000$ is equal to 647.8 €, which is slightly smaller than 666.31 € (the optimum solution value for P1 with flexible amortizations and $i_c = 1,000$). Concerning the flexible amortizations plan previously discussed, Graphic 9a presents the capital borrowed in short-term loans ($y_t$) and the total amount of capital in debt (excluding the long-term debt) in each period, while Graphic 9b shows the amortizations plans for all short-term borrowings ($x_{t+j}$ , $j=1,2,3$). Graphics 10a and 10b report the same information for the solution with the classical plan.

![Graphic 10: Flexible plan: (a) Short-term borrow $y_t$ and the total amount in debt (excluding the long-term debt) in each period; (b) amortizations plans for the short-term borrowings $x_{t+j}$, $j=1,2,3$.](image)
The information in Graphics 10 and 11 indicates that the flexible amortizations plan’s solution makes fewer calls for short-term loans than the classical plan. It also shows that the total amount of capital in debt in each period is basically the same along the time. The last period gap on capital in debt is closed at the end of the period.

Comparing the two solutions’ objective function values, the difference is 18.51 €, which may suggest that the proposed flexibility can be less helpful than expected, at least for the parameters’ values considered in the example. However, the flexibility for paying the short-term loans may still improve the best solutions returned by the fixed methodology, especially when cash availability is narrow. Anyway, although the flexible strategy may help improving cash availability, it seems that an adequate production plan and hiring strategy, combined with an appropriate allocation of financial means, may constitute the key elements for being successful.

So far, we have assumed that cash balance variables ($b_t$) will only assume non-negative values, forcing the set of solutions to avoid shortfalls. As mentioned in Section 3, this condition can be argued to be unreal, suggesting that the variables should assume negative values, as well. In order to analyze this aspect, we take again the previous discussion on the $i_c$ parameter values, while allowing variables $b_t$ to vary in the interval $[-1,000; +\infty[$, in all periods. This suggests a feasible short-fall not higher than 1,000 €. This case involves more than a simple variation on a given parameter. It can be done by setting the following variables substitution: $b'_t = b_t - 1000$, for all $t=1,\ldots,ST$, with $b'_t \geq 0$. Graphic 12 and Table 6 update Graphic 9 and Table 5 data for the new conditions.
Graphic 12: Cash-balance optimum values for problems’ P1 scenarios with shortfalls, involving variations in parameter $i_{c0}$.

<table>
<thead>
<tr>
<th>$i_{c0}$ values</th>
<th>$b_{xt}$</th>
<th>max</th>
<th>num. of workers</th>
<th>short-term loans: starting periods</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$b_{xt}$</td>
<td>1$^{st}$ year</td>
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</tr>
<tr>
<td>100</td>
<td>-229.71</td>
<td>6</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>1,000</td>
<td>698.93</td>
<td>6</td>
<td>7</td>
<td></td>
</tr>
<tr>
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</tr>
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<td>5,000</td>
<td>4,759.79</td>
<td>6</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>10,000</td>
<td>9,779.86</td>
<td>6</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>20,000</td>
<td>19,779.86</td>
<td>6</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

Table 6: Problem P1 alternative scenarios with shortfalls, involving variations in parameter $i_{c0}$.

As expected, the optima have improved for the scenarios with $i_{c0}$ equal to 1,000; 2,500 and 5,000 €. These were the cases that include cash-balance values equal to zero, in the previous analysis, as seen in Graphic 9. However, these improvements were smaller than 33 €, which is somehow disappointing. In addition, the solutions decreased the need for borrowing short-term loans. Also, the former impossible scenario involving $i_{c0} = 100$ becomes feasible, with a final cash-balance under default. Anyway, we should note that to concede shortfalls with no penalties is the same as getting tax-free loans. This is probably “unhealthy” for the entire economic environment, compared with the benefits that a company can afford to. We have also tested a more relaxed lower bound, in which shortfalls are not higher than 5,000 €. In this case, the best improvement is smaller than 94 € and short-term loans were less borrowed.

A final group of scenarios to discuss involves variations on the growth rate of the demand, considering $d_{pt} = (1.01)d_{pt-1}$, $d_{pt} = (1.02)d_{pt-1}$, $d_{pt} = (1.03)d_{pt-1}$ and $d_{pt} = (1.05)d_{pt-1}$, for $t = 2, ..., S T$, with $d_{p1} = 1000$ units, being represented by the following growth rates $\alpha = 1\%$, $2\%$, $3\%$ and $5\%$, respectively. Graphics 13a, 13b and 13c describe the capacity of
production, workforce availability, production level and stock level, in each period, for problem P1 under the three scenarios: $\alpha = 2\%$ (Graphic 13a), $\alpha = 3\%$ (Graphic 13b) and $\alpha = 5\%$ (Graphic 13c). Table 7 reports the optimum solution’s objective function values, the hiring strategies and the periods when short-term loans are borrowed. When $\alpha = 1\%$, the problem is impossible.

**Graphic 13:** Capacity of production, workforce capacity, production level and stock optimum values for problem P1 with $\alpha = 2\%$ (Graphic 13a), $\alpha = 3\%$ (Graphic 13b) and $\alpha = 5\%$ (Graphic 13c).

<table>
<thead>
<tr>
<th>$\alpha$ (%)</th>
<th>max $b_{st}$</th>
<th>num. of workers 1st year</th>
<th>2nd year</th>
<th>short-term loans: starting periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>impossible</td>
<td>6</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3634.93</td>
<td>6</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>9,779.86</td>
<td>6</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>12,690.03</td>
<td>7</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

**Table 7:** Problem P1 alternative scenarios, involving variations in the rates ($\alpha$) associated to parameter $dp_t$.

In the first scenario, with $\alpha = 2\%$, the demand is low, so the production is mostly below the workforce availability. Yet, the best hiring strategy in this case still involves 6 workers in the first year and 7 workers in the second one. Then, when $\alpha = 3\%$, there is no change in the hiring strategy, and the production level is better suited to the workforce availability. The last scenario, involving a faster growth of the demand, the solution requires 7 workers in the entire range, and the production uses most of the workforce availability. This scenarios sequence also reveals a growing usage of stock in order to support the last periods’ sales. In effect, in the lower rate scenario (with $\alpha = 2\%$), the production is mostly in line with the demand; with very low usage of stock to support the last period’s needs. Conversely, in the more demanding scenario, with the higher growth rate ($\alpha = 5\%$), the production planning makes a strong usage of stock at the early stages in order to meet the last periods stronger
demand. In this case, the strong usage of stock can help avoiding the need to hire an additional worker in the second year.

Short-term loans were only borrowed in the smaller growth demand scenario ($\alpha = 2\%$). This case is probably less profitable, which may force the solution to resort to this kind of loans in order to sustain the business.

**Problem P4 alternative scenarios discussion:**
We could also discuss all the previous scenarios within problem’s P4 framework. Instead, we simply copy the first analysis, involving variations on the initial capital supplied by the shareholders, parameter $ic_0$. Then, we discuss other alternative modifications.

Considering the initial capital supplied by the shareholders (parameter $ic_0$), there are two aspects that we would like to discuss, namely:

i) if there is any chance to solve FP4 with lower values for $ic_0$ than those tested within P1 (note that problem P1 was observed to be impossible for $ic_0 = 100$); and

ii) it was somehow unexpected to observe the large amount of cash released right in the first period in problem’s P4 solution, in the initial example. Curiously, this amount is larger than the initial capital supplied by the shareholders!

For conducting this discussion, we propose the following values for $ic_0$: -7,000, -6,000, -5,000; -1,700 and 5,000 €. Note that $ic_0 = 10,000$ € in the initial example.

Graphic 14 presents the cash-flows released outwards (including dividends) (variable $d_t$) during the entire short-term range, for the proposed scenarios (this time we use a 3D representation to help the reading).

![Graphic 14: Cash-flows released outwards optimum values for problems’ P4 scenarios, involving variations in parameter $ic_0$.](image-url)
Table 8 reports the objective function optimum values (the sum of all cash-flows released), the number of workers to hire and the periods in which the short-term loans were borrowed.

<table>
<thead>
<tr>
<th>$ic_0$ values</th>
<th>optimum value</th>
<th>num. of workers</th>
<th>short-term loans: starting periods</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1st year</td>
<td>2nd year</td>
</tr>
<tr>
<td>-7,000</td>
<td>impossible</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-6,000</td>
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<td>7</td>
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<tr>
<td>-5,000</td>
<td>35,090.85</td>
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<td>7</td>
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<td>38,404.86</td>
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<td>7</td>
</tr>
<tr>
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<td>45,104.86</td>
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<td>7</td>
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<tr>
<td>10,000</td>
<td>50,104.86</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 8: Problem P4 alternative scenarios, involving variations in parameter $ic_0$.

Concerning observation i), the more flexible cash release scheme proposed in problem P4 can assist the overall financial stream when the initial capital supplied by the shareholders becomes narrower. In fact, problem P4 was able to solve the problem with an initial deficit of 6,000 €, while P1 was unable to do it with an initial capital of 100 €. Furthermore, the sustainability condition forces the last period cash-balance ($b_{ST}$) to be at least 12,175 € (concerning the number of workers hired in the second year and the long-term loan payments), which is higher than the same period cash-balance in the optimum solution of FP1 ($b_{ST} = 9,779.86 €$), for the initial example. In addition, the total cash-flows released outwards in problem P1 (in the initial example) is equal to 45,000 €, using $ic_0 = 10,000 €$, while problem P4 can do better (45,104.86 €) with a smaller initial capital ($ic_0 = 5,000 €$). So, we can conclude again that the more flexible strategy proposed in problem P4 provides a better cash-flow adjustment for managing the entire process.

On the other hand, concerning observation ii), we saw in the initial example optimum solution of FP4 (in Subsection 4.1) that the first period cash-flow released outwards (variable $b_1$) is equal to 11,700 €, while the initial capital supplied by the shareholders ($ic_0$) is 10,000 €. This suggests that this capital is needless in the entire solution framework in problem P4 (with the initial example parameters’ values). In effect, if we decrease the initial capital to $ic_0 = 5,000 €$, the optimum solution value (sum of all cash-flows released) also decreases in the same amount. The same holds if the reduction is the entire quantity 11,700 € (released in the first period), by considering $ic_0 = -1,700 €$, meaning that we start with a 1,700 € deficit. Following this trend, if the reduction is higher, the resulting effect in the objective function value is no longer proportional, as seen in the scenarios with $ic_0 = -5,000$ and -6,000 €. These observations reinforce that the capital released right in the first period is
Another relevant parameter to discuss is the sales unitary net profit (parameter \( np_t \)) that was considered to be equal to 8 €/unit in the initial version discussed in Subsection 4.1. The alternative scenarios involve greater and smaller values for parameter \( np_t \), considering \( np_t = 6, 7, 9 \) and 10 €/unit, besides the previous case with \( np_t = 8 \) €/unit. Graphic 15 presents the cash-flows released outwards (including dividends) (variable \( d_t \)) during the entire short-term range, for the proposed scenarios.

![Graphic 15: Cash-flows released outwards optimum values for problems’ P4 scenarios, involving variations in parameter \( np_t \).](image)

Table 9 reports the objective function optimum values (the sum of all cash-flows released), the number of workers to hire and the periods when cash-flows are released.

<table>
<thead>
<tr>
<th>( np_t ) values</th>
<th>optimum value</th>
<th>num. of workers</th>
<th>periods when cash-flows are released</th>
</tr>
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<tbody>
<tr>
<td>6</td>
<td>5,550.08</td>
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<td>1, 2, 3, 4, 5, 6, 7, 8, 9, 10</td>
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<td>7</td>
<td>27,813.96</td>
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<td>1, 2, 3, 4, 5, 6, 7, 8, 9, 10</td>
</tr>
<tr>
<td>8</td>
<td>50,104.86</td>
<td>6, 7</td>
<td>1, 2, 3, 4, 5, 6, 7, 8, 9, 10</td>
</tr>
<tr>
<td>9</td>
<td>72,512.65</td>
<td>6, 7</td>
<td>1, 2, 3, 4, 5, 6, 7, 8, 9, 10</td>
</tr>
<tr>
<td>10</td>
<td>94,942.65</td>
<td>6, 7</td>
<td>1, 2, 3, 4, 5, 6, 7, 8, 9, 10</td>
</tr>
</tbody>
</table>

Table 9: Problem P4 alternative scenarios, involving variations in the sales unitary net profit (parameter \( np_t \)).

The results indicate an expected evolution of the cash-flows released, when the sales profits increase. Also expected, month 12 is spared to this process, naturally due to the effort to pay the long-term loan’s amortization and interest rates. However, it is somehow unexpected to observe that the various cash released strategies have no common pattern. In effect, there are
periods in which the more profitable sales schemes release fewer cash-flows than some of
the less profitable sales solutions. Moreover, the hiring strategy doesn’t change and there is
no need for short-term loans, in all these cases.

A final analysis involves incorporating inflation in problem P4 context. This is a relevant
concern for countries under high inflation rates, as argued in Kirca & Köksalan (1996). For
this purpose, we suggest analyzing four values for the annual inflation rate: \( \mu = 0.03, 0.05,
0.1 \) and \( 0.2, \) besides the original problem version in which the inflation effect was ignored
\( (\mu = 0). \) Then, we select a number of parameters that can be sensitive to inflation, namely
the cost per worker (parameters \( c_t \)), the production fixed cost (parameters \( f_c \)), the unitary cost
for stocking (parameters \( s_c \)) and the interest rate for the short-term loans (parameters \( s_r \)).
We assume that the workers’ salaries are updated only once a year. This implies that
parameter \( c_t \) is only modified in month 13, within the planning horizon under discussion.
All the other parameters are updated monthly, regarding the proposed annual inflation rates.
We could also incorporate inflation in the sales net profit (parameter \( n_p \)). However, we
chose not to do it, assuming that the production costs and sales prices cancel each other the
inflation effect, keeping the net profit in balance.

Another relevant aspect to consider involves the overall financial market activity, which
usually accelerates when the inflation rate grows. This acceleration can bring new financial
products into the market with stimulating interest rates, carrying the chance for the company
to allocate unused cash-flows to those new financial products. In order to simulate this
aspect, we consider the mentioned unused cash-flows are represented by the extra cash-
balance, being characterized by the difference between two successive cash-balances, that is,
\( (b_t - b_{t-1}) \), for period \( t \), if the result is positive. So, the extra cash-balance is represented by
the following continuous variable

\[
v_t = \max (0, b_t - b_{t-1}) , \text{ for } t = 1, \ldots, ST-1 \text{ with } b_0 = ic_0 \tag{13}
\]

We assume that each period \( t \) extra cash-balance (variable \( v_t \) value) is invested in the
mentioned new financial products, made available by banks, for instance. The interest rate
returned by these financial products is also influenced by the inflation, being updated in each
month (period), as well, according to the inflation rate.

To represent condition (13), we introduce the sets of constraints (14)-(16) and the new set of
binary variables \( z_t \), taking value 1 if \( b_t > b_{t-1} \); and 0, otherwise.
\[
\begin{align*}
\nu_t & \leq b_t - b_{t-1} + M (1 - z_t), \quad t = 1, \ldots, ST-1 \\
\nu_t & \leq M \cdot z_t, \quad t = 1, \ldots, ST-1 \\
b_t - b_{t-1} + m & \leq M \cdot z_t, \quad t = 1, \ldots, ST-1 \\

\nu_t & \geq 0, \quad z_t \in \{0, 1\}, \quad t = 1, \ldots, ST-1
\end{align*}
\] 

In this representation, \( M \) is a sufficiently large constant (say, 100,000) and \( m \) is a sufficiently small constant (say, 0.001). In addition, if we denote by \( ir_t \) the parameter that represents the interests net rate for an investment made in period \( t \), then \( (ir_t \cdot \nu_t) \) is the net capital received for an invested amount of \( \nu_t \), which becomes a cash in-flow in period \( t+1 \). To represent this aspect, we add the term \( (ir_t \cdot \nu_t) \) to the right-hand-side of the associated equality (7b', 7c', 7d' or 7e'), for period \( (t+1) \).

Thus, problem P4 that incorporates the previously described inflation effects can be represented by model FP4 augmented with constraints (14)-(17) and the mentioned modification in equalities (7b', 7c', 7d' or 7e'), being denoted by FP4'.

Considering the initial problem data in which inflation was ignored (\( \mu = 0 \)) and in order to discuss the new inflation scenarios, involving the annual inflation rates \( \mu = 0.03, 0.05, 0.1 \) and 0.2, we have adjusted most of the starting values for the interest rates used in the model. These values are shown in Table 10, addressing just the first month data. As mentioned before, all the remaining periods’ values are updated according to the inflation effect. The table also includes the first month interest rate associated to the extra cash-balance invested (parameter \( ir_1 \)). We further recall that all loan’s interest rates include taxes (\( sr_1, sr_t \) and \( lr \)), while the investment interest rate \( (ir_t) \) is a net value, incorporating no taxes. Also, \( lr \) is an annual rate, while all the remaining parameters are monthly rates.

The additional sets of constraints introduced in FP4 inhibit the use of Excel’s build-in solver for answering model FP4’. Alternatively, we used both lp_solve and CPLEX packages for solving it. The three last columns in Table 10 report the associated information, namely, the optimum solution value and the times (in seconds) required by the two solvers to reach the optimums.
As expected, the inflation effect deteriorates the objective function value (sum of all cash-flows released). Once again, the hiring strategy doesn’t change and there is no need for short-term loans, in all these cases. Also, contrarily to the initial example involving P4 (case $\mu = 0$), all cash-flows released outwards (including dividends) are paid just in the last period (variable $d_{ST}$). In effect, this time the company has the chance to profit from alternative investments. Then, instead of releasing the capital, the company uses it to its own profit.

Another aspect to focus on is the relationship between the investment’s interest rates ($ir_t$) and the short-term loan’s interest rates ($sr_t$), where $sr_t$ was chosen to be below $ir_t$, in all scenarios. This choice was to avoid the chance of using short-term borrows for investing in alternative financial products. In effect, and as expected, this was observed in a number of preliminary scenarios in which $sr_t > ir_t$.

- Concerning the execution times of the two solvers, we note that CPLEX is much faster to reach the optimum, in all these cases. Yet, the lp_solve package is cost free, which may constitute a relevant aspect to consider for a small or medium sized company.

**Other scenarios:**

The discussion conducted so far involved a number of potential scenarios, leaving aside many more alternative discussions, some of those we have actually tried but didn’t include in the paper, namely:

i) larger values for the short-term loan’s State rate (parameter $sr_2$), which reduces the real rate applied to those loans (defined by $sr_t$), representing a larger State support; and

ii) more demanding requirements for the sustainability condition in problem P4, forcing more than a single month’s salaries to be guaranteed.

Other discussions that we haven’t tested, but with promising interesting insights, namely:
iii) the inclusion of a profit rate associated to cash-balance variables, representing alternative financial investments, as suggested in Subsection 4.1 in problem’s P2 analysis;
iv) the inclusion of payment delays, as suggested in Zeballos et al. (2013); and
v) other aspects involving the production process, namely, multi-item, multi-level, fixed costs, backlogging, different types of workers with different salaries and other aspects; as considered in a large number of lot-sizing papers in the literature.

To conclude, and as mentioned before, we should let the decision maker conduct his/her own course, using the mathematical model as a tool for answering his/her own questions and doubts, by simulating adequate scenarios and analyzing the answers.

5. Conclusions

Planning production and workforce is a growing theme in real-world industry and research. It is also a matter of consensus that these subjects should run together with financial planning concerns. In effect, this joint framework has been capturing the attention of a growing number of researchers and company’s managers.

This paper is intended to contribute for the mentioned discussion, addressing a specific problem that also puts in a single framework: production, workforce sizing and financial planning requirements. In this case, the problem involves a single product on a single-level production scheme, covered by a single type of workers. The financial process includes a long-term loan, in which amortizations are made in equal amounts; and an innovative scheme of three months loans for helping paying salaries. These short-term loans cover the entire time-horizon under discussion and amortizations are made at the best convenience of the company. The short-term periods for these loans can reduce the risk of default, which may strengthen bank’s confidence for allowing the associated flexible amortizations plans.

Under these guidelines, we have proposed four problems, to which general formulations were introduced. Then, we presented a specific case study (with fictitious data) and used it for discussing the problems, their optimum solutions and alternative scenarios, involving parameters variations and other modifications in the models.

The solutions revealed very accurate strategies for managing the entire planning streams, contributing for reaching the most profitable gains, according to the objective functions involved. They also showed that when giving the problem the chance to release cash-flows (including dividends) at the best convenience of the company, the solutions become more
profitable, showing that the mentioned flexibility can truly aid the overall financial process. Furthermore, when cash availability is narrower, the solutions propose borrowing short-term loans, which revealed to be crucial for avoiding shortfalls.

Still, we can argue that real-world problems include many other variables and features that were not comprised in these models, suggesting that the rigorous solutions obtained can easily fall apart from reality, when the time starts running. In effect, we shouldn’t ignore that we are planning over the future. However, we also stress the importance for analyzing and discussing the problems and the various scenarios that we suspect to be more sensitive, for decision making support. Then, when the time starts running, we should compare the two processes (solution planned and reality) at all time, as suggested when dealing with project planning. Hence, if the observed solution starts to move apart from the initial plan, we can still use the model for readjusting the remaining process, by making adequate changes in the parameters and fixing the variables to which the time is done.

The proposed models are intended to encourage a more regular usage of mathematical programming based tools in order to provide companies’ managers with the possibility of conducting their own solutions analysis and alternative scenarios discussions. For this reason, we conducted the discussion trying to simulate a decision maker oriented analysis, assuming that the decision maker has some knowledge of the environmental behavior of the problem in hands. The more complex approaches can be complemented in collaboration with trained professionals on mathematical programming, allowing the managers to focus on the companies concerns, by analyzing solutions and discussing appropriate scenarios.

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**References**


