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Maria João Cortinhal\textsuperscript{a,b*}, Anabela Costa\textsuperscript{a,b}, Maria João Lopes\textsuperscript{a,b}, Ana Catarina Nunes\textsuperscript{a,b}

\textsuperscript{a} University Institute of Lisbon , (ISCTE-IUL), Avenida das Forcas Armadas 1649-026 Lisbon, Portugal
\textsuperscript{b} CIO, Faculdade de Ciências, Universidade de Lisboa, 1716-049 Lisboa, Portugal

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Abstract

This research extends the work in the field of logistic network design by introducing a new mixed integer linear programming model to solve an integrated location, production and distribution planning problem in a multi-plant, multi-item, multi-retailer, multi-modes of transportation and single period logistic environment. Moreover, it also models situations on which organizations can opt for outsourcing as a strategic way to meet all the customer demands. We propose several valid inequalities to strengthen the LP relaxation of the model. Computational experiments were conducted on a set of randomly generated instances. It was possible to conclude that one set of the proposed valid inequalities led to significant improvements.

Keywords: Logistics, Location, Facilities planning and design, Integer programming, Linear programming.

1 Introduction

Nowadays, changeable economic conditions and supply chain dynamics are obliging manufacturing and distribution companies to turn to network design as a solution to remain relevant in the global marketplace. Factors such as wide variety of products, global markets, and more demanding customers, among others, are obliging companies to deal with more and more complex supply chain networks. Furthermore, as pointed out by [9], manufacturing companies increasingly need to integrate production and transport planning in order to optimize both these processes simultaneously.

There is a vast literature about network supply chain networks. Some of them rely on general models, e.g. [10], whereas others deal with real world applications, e.g. [11]. The

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interested reader is referred to some recent reviews, such as [9], concerned with mathematical programming models for supply chain production and transport planning, and [8] more focused on facility location analysis within the context of supply chain management.

This research extends the work in the field of logistic network design problems by introducing a new model. Additionally to location and modular capacity choices for production centers and warehouses, suppliers and multiple modes of transportation, product range assignment and product flows, this model incorporates minimum levels of service and outsourcing.

Many public and private organizations are using outsourcing as a way to improve their effectiveness and efficiency. Despite being commonly related with services, it can be used as an alternative way for the production planning, as well. As pointed out by [7], a company can face the problem of having limited production capacities using different strategies. Postponing the demand, such as in [1], or covering the demand by paying an extra cost through outsourcing are two of them.

Outsourcing strategies have already been referred in the literature related with production planning. In [2] a production problem is formulated as a Linear Programming problem so as to maximize the throughput from manufacturing and from outsourcing products. In [6], and [7] a dynamic capacitated production planning problem with outsourcing is studied. They both consider a multi-period planning model where the customer demands are met by production or by outsourcing without postponement or backlog. [12] presents an analytical approach to investigate the optimal decisions on a multi-product and multi-period production planning problem with stochastic demands, in which there exists two alternatives for production: manufacturing all the parts of the products and then assemble them, or outsourcing parts of the products and then assemble them.

To the best of authors knowledge, a model for a logistic network design as the one considered in this research that also consider outsourced finished products as a way to cover the demand have never been addressed in the literature. Here we introduce a linear mixed integer programming formulation for this problem.

The contribution of this paper is to introduce a general model for a supply chain network design that integrates strategic and tactical decisions, and that allows outsourcing strategies as a way to cover the demand.

Furthermore, we propose valid inequalities to strengthen the LP relaxation of the model some of them yielding significant improvements.

The rest of the paper is organized as follows. Section 2 is devoted to the description and the formulation of the problem. Valid inequalities for the problem are presented in Section 3. The random generation of the instances as well as the computational experimentations are addressed in Section 4. In Section 5 conclusions, remarks and future work are presented.
2 Problem definition

In this section we describe the general framework for the logistic network under study. Moreover, the mathematical formulation for the problem is presented.

2.1 General framework

The integrated intermodal supply chain network design here described involves a set of suppliers $S$, a set of raw materials $R$, a set of products $P$, a set of site locations for production centers $L$, a set of site locations for warehouses $W$, a set of customers $C$, and a set of site location sizes $T$.

Each supplier $s \in S$ is eligible to provide a subset of raw materials, which in turn are used in predetermined quantities to manufacture products $p \in P$.

For each potential site to locate production centers, where the production occurs, modular production capacities are considered. Production capacities imposes upper bounds on the total production and on per product production. The fixed set-up and operational costs for production centers are capacity dependent whereas the production costs are variable dependent.

Once manufactured, products can be delivered either directly to customers or to warehouses, where they are stored and then sent to customers. Therefore, production centers and warehouses can store finished products, and so for both, production centers and warehouses, total and per product storage capacities are considered.

For warehouses $w \in W$, location and sizing choices, as well as fixed set-up and operational costs are also considered.

Deliveries take place using multiple modes of transportation. Thus, more than one mode of transportation $m \in M$ can be selected for delivering items (raw materials and finished products) between suppliers and productions centers as well as between production centers and warehouses or customers. Each mode of transportation incurs a fixed cost and a variable cost, which is in turn dependent on the distance and on the item, namely its quantity.

Additionally, a minimum level of service is defined for both production centers and warehouses. This minimum level of service guarantees that if one facility (plant or warehouse) is operating it will have to fulfill a certain percentage of its total capacity. For production centers, the minimum level of service is related with production capacities whereas for distribution centers is related with storage capacities.

It is assumed that demands are given and fixed. To cover the demand, warehouses can also be supplied with finished products coming from external suppliers. However, the amount of outsourced products cannot exceed a pre-defined percentage of the total demand. Outsourced...
products have a penalty cost assigned to, which is product dependent.

2.2 Mathematical model

We now provide a mathematical mixed integer programming (MIP) formulation of the problem. Since the formulation requires the use of several notation, to make things easier, the whole notation is summarized in Table 6, given in A.

Let us consider the following binary decision variables:

\[ Z_t^l = \begin{cases} 1, & \text{if a plant of size } t \text{ is located at site } l, \ l \in L, t \in T \\ 0, & \text{otherwise} \end{cases} \]

\[ Z_t^w = \begin{cases} 1, & \text{if a warehouse of size } t \text{ is located at site } w, \ w \in W, t \in T \\ 0, & \text{otherwise} \end{cases} \]

and \( Y_{od}^m \) that will be 1 if mode of transportation \( m \) is selected for deliveries from origin \( o \) to destination \( d \), \( m \in M, o \in O, d \in D \), and 0 otherwise.

Additionally, let \( X_{od}^{im} \) be the amount of item \( i \) delivered from origin \( o \) to destination \( d \), \( i \in I, m \in M, o \in O, d \in D \), and \( X_{lp} \) be the amount of product \( p \) provided by an external supplier to a warehouse \( w \), \( w \in W, p \in P \).

The problem can be formulated as a MIP as follows:

\[
(M) \ \text{Min} \ \sum_{t \in T} \left[ \sum_{l \in L} \left( f_1^t + f_3^t \right) Z_t^l + \sum_{w \in W} \left( f_1^w + f_3^w \right) Z_t^w \right] \\
+ \sum_{m \in M} \sum_{l \in L} \sum_{s \in S} f_2^{tm} Y_{sl}^m \\
+ \sum_{r \in R} \sum_{s \in S} \sum_{l \in L} \sum_{m \in M} \left( c_1^r + \text{dist}_{sl} c_2^m \right) X_{sl}^{rm} \\
+ \sum_{m \in M} \sum_{l \in L} \sum_{w \in W} \sum_{p \in P} f_2^{wm} Y_{lw}^{pm} + \sum_{p \in P} \text{dist}_{lw} c_2^m X_{lw}^{pm} \\
+ \sum_{m \in M} \sum_{l \in L} \sum_{c \in C} f_2^{mc} Y_{lc}^{pm} + \sum_{p \in P} \text{dist}_{lc} c_2^m X_{lc}^{pm} \\
+ \sum_{m \in M} \sum_{w \in W} \sum_{c \in C} f_2^{wc} Y_{wc}^{pm} + \sum_{p \in P} \text{dist}_{wc} c_2^m X_{wc}^{pm} \\
+ \sum_{l \in L} \sum_{m \in M} \sum_{w \in W} c_3^l \left( X_{lw}^{pm} + X_{lc}^{pm} \right) \\
+ \sum_{w \in W} \sum_{p \in P} P c p X E_w^p \right) 
\]
\[
\sum_{m \in M^r_{sl}} \sum_{l \in L} X'^m_{sl} \leq ARM^r_s, \quad r \in R, s \in S^r \\
\sum_{r \in R} CC'^m X'^m_{sl} \leq CM^m X'^m_{sl}, \quad m \in M^r_{sl}, l \in L, s \in S \\
CC'^m X'^m_{sl} \leq CMP^m X'^m_{sl}, \quad r \in R, m \in M^r_{sl}, l \in L, s \in S^r \\
\sum_{s \in S} \sum_{m \in M^r_{sl}} X'^m_{sl} = \sum_{p \in P^r} a'^p \left( \sum_{w \in W} \sum_{m \in M^r_{lw}} X'^{pm}_{lw} + \sum_{m \in M^r_{lc}} \sum_{c \in C^p} X'^{pm}_{lc} \right), \quad r \in R, l \in L \\
\sum_{m \in M} CPC^p \left( \sum_{w \in W^p} X'^{pm}_{lw} + \sum_{c \in C^p} X'^{pm}_{lc} \right) \leq \sum_{t \in T} APC^p Z'^t_l, \quad p \in P, l \in L^p \\
\sum_{p \in P} \sum_{m \in M} CPC^p \left( \sum_{w \in W^p} X'^{pm}_{lw} + \sum_{c \in C^p} X'^{pm}_{lc} \right) \leq \sum_{t \in T} AP^p Z'^t_l, \quad l \in L \\
\sum_{p \in P} \sum_{m \in M} CPC^p \left( \sum_{w \in W^p} X'^{pm}_{lw} + \sum_{c \in C^p} X'^{pm}_{lc} \right) \geq \sum_{t \in T} \alpha^t_l AP^p Z'^t_l, \quad l \in L \\
CC'^{pm} X'^{pm}_{lw} \leq CM'^{pm}_{lw} Y'^{pm}_{lw}, \quad m \in M^p_{lw}, w \in W, l \in L \\
CC'^{pm} X'^{pm}_{lw} \leq CM'^{pm}_{lw} Y'^{pm}_{lw}, \quad p \in P, m \in M^p_{lw}, l \in L^p, w \in W^p \\
CC'^{pm} X'^{pm}_{lc} \leq CM'^{pm}_{lc} Y'^{pm}_{lc}, \quad m \in M^p_{lc}, l \in L, c \in C \\
CC'^{pm} X'^{pm}_{lc} \leq CM'^{pm}_{lc} Y'^{pm}_{lc}, \quad p \in P, m \in M^p_{lc}, l \in L^p, c \in C^p \\
\sum_{m \in M} \sum_{w \in W^p} X'^{pm}_{lw} + \sum_{c \in C^p} X'^{pm}_{lc} \leq \sum_{t \in T} ASC^p Z'^t_l, \quad p \in P, l \in L^p \\
\sum_{p \in P} \sum_{m \in M} \left( \sum_{w \in W^p} X'^{pm}_{lw} + \sum_{c \in C^p} X'^{pm}_{lc} \right) \leq \sum_{t \in T} AS^t_l Z_l, \quad l \in L \\
\sum_{t \in T} Z_l \leq 1, \quad l \in L \\
\sum_{p \in P} CC'^{pm} X'^{pm}_{wc} \leq CM'^{pm}_{wc} Y'^{pm}_{wc}, \quad m \in M^p_{wc}, w \in W, c \in C \\
CC'^{pm} X'^{pm}_{wc} \leq CM'^{pm}_{wc} Y'^{pm}_{wc}, \quad p \in P, m \in M^p_{wc}, w \in W^p, c \in C^p \\
\sum_{m \in M^p_{wc}} \sum_{c \in C^p} X'^{pm}_{wc} \leq \sum_{t \in T} ASC^p_{wc} Z^t_{wc}, \quad p \in P, w \in W^p \\
\sum_{p \in P} \sum_{m \in M^p_{wc}} \sum_{c \in C^p} X'^{pm}_{wc} \leq \sum_{t \in T} AS^t_{wc} Z^t_{wc}, \quad w \in W \\
\sum_{p \in P} \sum_{m \in M^p_{wc}} \sum_{c \in C^p} X'^{pm}_{wc} \geq \sum_{t \in T} \alpha^t_{wc} AS^t_{wc} Z^t_{wc}, \quad w \in W 
\]
\[
\sum_{t \in T_w} Z_t^I \leq 1, \quad w \in W \tag{21}
\]

\[
\sum_{m \in M^p_{lw}, l \in L^p} X^{pm}_{lw} + \sum_{m \in M^p_{lc}, w \in W^p} X^{pm}_{lc} = d^p_t, \quad p \in P, c \in C^p \tag{22}
\]

\[
\sum_{m \in M^p_{lw}, l \in L^p} X^{pm}_{lw} = \sum_{m \in M^p_{wc}, c \in C^p} X^{pm}_{wc} - X E^p_w, \quad p \in P, w \in W^p \tag{23}
\]

\[
\sum_{p \in P} \sum_{w \in W^p} X E^p_w \leq \beta \sum_{p \in P} \sum_{c \in C^p} d^c_p \tag{24}
\]

\[
X_{sl}^r, X_{lw}^r, X_{lc}^r, X_{wc}^p \geq 0, \quad s \in S, l \in L, w \in W, c \in C, m \in M, p \in P, r \in R \tag{25}
\]

\[
X E^p_w \geq 0, \quad w \in W, p \in P, r \in R \tag{26}
\]

\[
Y_{sl}^m, Y_{lw}^m, Y_{lc}^m, Y_{wc}^m \in \{0, 1\}, \quad s \in S, l \in L, w \in W, c \in C, m \in M \tag{27}
\]

\[
Z_t^l, Z^t_w \in \{0, 1\}, \quad l \in L, t \in T \tag{28}
\]

for given \(a_t^l, a_w^t \in [0, 1], l \in L, w \in W, t \in T\) and \(\beta \in [0, 1]\). Note that the \(a_t^l, k \in K\) parameters allow to define the minimum levels of service whereas the \(\beta\) is necessary to impose an upper bound on the amount of outsourced products.

The objective function contains components related with fixed and operational costs for production centers and warehouses, acquisition costs, fixed costs and variable transportation costs for raw materials, fixed and variable transportation costs for products, production costs for products and penalty costs.

Constraints (2), (3) and (4) are related with suppliers: constraints (2) limit the amount of raw materials provided per supplier whereas constraints (3) and constraints (4) are total and per raw material capacity constraints for the modes of transportation.

Constraints (5) guarantee that each production center will receive enough raw material for its own production.

Constraints (6) to (15) impose restrictions to plants. Constraints (6) and (7) are per product and total production capacity constraints whereas constraints (8) impose minimum levels of production. Constraints (9) to (12) are global and per product capacity constraints on transportation modes that depart from plants whilst constraints (13) and (14) are per product and global storage capacity constraints. Finally, constraints (15) establish that no more than one size may be selected at each site location for production centers.

Constraints (16) to (19) and constraints (21) have the same meaning of constraints (11) to (15) but for warehouses instead of being for plants whereas constraints (20), which are similar to constraints (8), impose minimum levels of storage.

Constraints (22) ensure that customer demands are fulfilled whereas constraints (23) guarantee that the goods received at each warehouse are delivered to customers.
Constraint (24) ensures that no more than a 100\% percentage of the total demand can be fulfilled with outsourced finished products. To end up, constraints (25) to (28) are the sign constraints.

Note that model (1)–(28) assumes that there exists a fixed cost for each mode of transportation. If not applied, the former model can be rewritten without the $Y_{sl}^m$ binary variables. In this case, it is enough to remove the $Y_{sl}^m$ variables from each constraint where they appear, and so their coefficients will be the new right hand sides. Thus, the resulting model, hereafter named as M1, will have the same number of constraints but less $\vert M \vert \vert L \vert (\vert W \vert + \vert C \vert) + \vert W \vert \vert C \vert$ variables.

3 Valid inequalities

In this section we present valid inequalities that can help to strengthen the LP relaxation of the proposed model.

3.1 Relating location and transportation modes variables

The formulation proposed in the previous section involves several sets of binary variables. Some of these sets are related. For instance, if raw material $r$ is delivered to plant $l$ by supplier $s$ then at least one mode of transportation $m$ between $s$ and $l$ must be selected. Thus, $Y_{sl}^m = 1$ for at least one $m$. Furthermore, plant $l$ will be used for production, and so $\sum_{t \in T} Z_t^l = 1$. Therefore, values that variables $Y_{sl}^m$ can take on are a lower bound on $\sum_{t \in T} Z_t^l$.

However, none of the constraints tie different sets of binary variables. In this section we propose several valid inequalities tying the sets of binary variables associated with delivering ($Y$ variables) with the sets of binary variables for location choice of production centers or location choice of warehouses ($Z$ variables).

The first set of valid inequalities, which relates suppliers with production centers, is as follows:

$$Y_{sl}^m \leq \sum_{t \in T} Z_t^l, \quad s \in S, \ l \in L, \ m \in M$$

Constraints (29) ensure that the mode of transportation $m$ is not used for deliveries between supplier $s$ and a production center located at $l$ unless at least one type of production center is opened at $l$.

**Proposition 1** (29) is valid for the problem.

**Proof:** Consider two cases: a) $\sum_{t \in T} Z_t^l = 1$ and b) $\sum_{t \in T} Z_t^l = 0$. 

7
Case a): if $\sum_{t \in T} Z_{lt}^t = 1$, then (29) becomes $Y_{sl}^m \leq 1$, which is valid, due to constraints (27);

Case b): if $\sum_{t \in T} Z_{lt}^t = 0$, then $\sum_{p \in P} \sum_{m \in M} \left( \sum_{w \in W} X_{lw}^{pm} + \sum_{c \in C} X_{lc}^{pm} \right) = 0$, $l \in L$, due to constraints (7). Furthermore, due to constraints (5), $\sum_{r \in R} \sum_{s \in S} \sum_{m \in M_{sl}} X_{rs}^m = 0$, $l \in L$. So, $Y_{sl}^m = 0$ for the optimal integer solution, since $f_{2}^{2m}$ is non-negative.

Other sets of valid inequalities can be derived considering other origin-destination pairs as follows:

$$Y_{lw}^m \leq \sum_{t \in T} Z_{lt}^t, \ l \in L, \ w \in W, \ m \in M \quad (30)$$

$$Y_{lw}^m \leq \sum_{t \in T} Z_{wt}^t, \ l \in L, \ w \in W, \ m \in M \quad (31)$$

$$Y_{wc}^m \leq \sum_{t \in T} Z_{wt}^t, \ w \in W, \ c \in C, \ m \in M \quad (32)$$

$$Y_{lc}^m \leq \sum_{t \in T} Z_{lt}^t, \ l \in L, \ c \in C, \ m \in M \quad (33)$$

We omit the proofs of validity for constraints (30), (31), (32), (33) since they are similar to the previous one.

### 3.2 Lower bound on the overall production capacity

Constraints that impose a lower bound on the number of facilities that must be opened on any feasible solution are frequently used as valid inequalities for discrete capacitated facility location models.

In order to evaluate this lower bound, only the maximum capacity available at each site location can be considered if size capacity choices are allowed. Moreover, in the proposed model outsourcing strategies are being considered.

It is then straightforward that these two conditions can unable the determination of effective lower bounds. Therefore, we opted for imposing a lower bound on the overall production capacity that is required to fulfill all the demand. This lower bound can be determined by solving a mixed integer problem, and takes into account the different capacity sizes.

Constraints (24) impose that no more than $100\beta$ percent of the overall demand can be covered by external suppliers, and so the total production capacities of the selected production centers must fulfill at least $100(1 - \beta)$ percent of the total demand.
Thus, a lower bound on the overall production capacity can be obtained by solving the following mixed integer problem:

\[
\begin{align*}
\text{Min } Z &= \sum_{l \in L} \sum_{t \in T} AP_l Z_t^l \\
\text{s.to: } \sum_{p \in P} \sum_{m \in M} CPC^p \left( \sum_{w \in W^p} X_{lw}^{pm} + \sum_{c \in C^p} X_{lc}^{pm} \right) &\leq \sum_{t \in T} AP_l Z_t^l, \ l \in L \\
\sum_{p \in P} \sum_{m \in M} CPC^p \left( \sum_{w \in W^p} X_{lw}^{pm} + \sum_{c \in C^p} X_{lc}^{pm} \right) &\geq \sum_{t \in T} \alpha_t^l AP_l Z_t^l, \ l \in L \\
\sum_{t \in T_l} Z_t^l &\leq 1, \ l \in L \\
\sum_{p \in P} \sum_{m \in M} CPC^p \left( \sum_{w \in W^p} X_{lw}^{pm} + \sum_{c \in C^p} X_{lc}^{pm} \right) &\geq (1 - \beta) \text{TotDem}, \ p \in P \\
X_{lw}^{pm}, X_{lc}^{pm} &\geq 0, \ l \in L, w \in W, c \in C, m \in M, p \in P \\
Z_t^l &\in \{0, 1\}, \ l \in L, t \in T
\end{align*}
\]

where \(\text{TotDem}\) is the overall demand. Let \(\text{LowerCap}\) be the optimum value of the former problem. Then, the constraint

\[
\sum_{l \in L} \sum_{t \in T} AP_l Z_t^l \geq \text{LowerCap}
\]

is a valid inequality for the model (1)–(28).

### 3.3 Strengthening production capacity constraints

Let us focus on production capacities on production centers and on available capacities at suppliers. By one side, constraints (6) and (7), which are per product and total production capacity constraints, impose an upper bound on the amount of finished products that can be produced at each production center. By the other side, constraints (2) to (5) limit the amount of raw materials that can be provided by each supplier.

Therefore, one can try to find out the maximum total production at each production center by solving the following mixed integer problem:

\[
\begin{align*}
\text{Max } Z_l &= \sum_{p \in P} \sum_{m \in M} CPC^p \left( \sum_{w \in W^p} X_{lw}^{pm} + \sum_{c \in C^p} X_{lc}^{pm} \right) \\
\text{s.to: } \sum_{m \in M_l^r} \sum_{l \in L^p} X_{ml}^{lm} &\leq ARM_s^r, \ r \in R, s \in S^r
\end{align*}
\]
\[
\sum_{r \in R} CC^r m X^r_{sl} \leq CM^r m Y^r_{sl}, \quad m \in M_{sl}, s \in S^r \quad (3)
\]
\[
CC^r m X^r_{sl} \leq CMP^r m Y^r_{sl}, \quad m \in M_{sl}, r \in R, s \in S^r \quad (4)
\]
\[
\sum_{s \in S} \sum_{m \in M_{sl}} X^r_{sl} = \sum_{p \in P} a^p \left( \sum_{w \in W} \sum_{m \in M_{lw}} X^p_{lw} + \sum_{m \in M_{lc}} \sum_{c \in C} X^p_{lc} \right), \quad r \in R \quad (5)
\]
\[
\sum_{m \in M} CPC^p \left( \sum_{w \in W} X^p_{lw} + \sum_{c \in C} X^p_{lc} \right) \leq APC^p_{lt_{\text{max}}}, \quad p \in P \quad (40)
\]
\[
\sum_{p \in P} \sum_{m \in M} CPC^p \left( \sum_{w \in W} X^p_{lw} + \sum_{c \in C} X^p_{lc} \right) \leq AP_{lt_{\text{max}}} \quad (41)
\]
\[
X^r_{sl}, X^p_{lw}, X^p_{lc} \geq 0, \quad s \in S, l \in L, w \in W, c \in C, m \in M, p \in P, r \in R \quad (42)
\]
\[
Y^m_{sl} \in \{0, 1\}, \quad s \in S, l \in L, m \in M \quad (43)
\]

where for each \( l \in L \), \( t_{\text{max}}^l \) stands for \( t \in T : AP_{lt_{\text{max}}} = \max_{t \in T} AP_{lt} \).

Let \( \text{CapMin}_l \) be the optimal value of the former problem. For each production center \( l \), \( \text{CapMin}_l \) is an upper bound on the total production capacity. Therefore, if \( \text{CapMin}_l \) is smaller than \( AP_{lt} \) for a given size \( t \), then one can replace the total capacity \( AP_{lt} \) by \( \text{CapMin}_l \) in the model (1)–(28). Hereafter, this new model will be referenced as M2.

### 4 Computational study

To evaluate the proposed models as well as the proposed valid inequalities, computational experiments on a set of randomly instances were done. In this section we present not only the methodology that was used for the random generation, but also the computational results.

#### 4.1 Generating instances

This research was based on a project undertaken for a Portuguese cement company. However, like most business organizations, this one does not publicly share their data. Moreover, it was not possible to find available instances due to the own characteristics of the proposed supply chain network design. Therefore, randomly generated instances was the only way left for testing the proposed model.

Since our models shares characteristics with the one presented in [3], we followed their methodology to randomly generate some of the parameters. Some exceptions were made not only because of the differences between the models, but also to try to incorporate some of the features of the integrated production and transportation model developed for the cement company.
To begin with, to represent suppliers, and potential sites to locate production centers and warehouses, we used \((x,y)\) coordinates. The \((x,y)\) coordinates were drawn from the uniform distribution: \(U[0,160] \times U[0,560]\). The range of the uniform distribution was chosen in such a way that it could simulate the area and the shape of the continental Portuguese territory. These coordinates are then used to compute the Euclidian distance between each origin-destination pair.

As pointed out in [3], instances vary according to size, to complexity, which is determined by the capacity structure and by the flow magnitude, and to cost structure.

Size involves parameters such as the number of customers \(|C|\), the number of suppliers \(|S|\), the number of raw materials \(|R|\), the number of products \(|P|\), and the number of potential sites to locate production centers \(|L|\) and warehouses \(|W|\). It is quite straightforward that the cardinality of these sets should be related, since more customers can mean more products, which in turn means more raw materials, suppliers, production centers and warehouses. Therefore, we considered \(|C| = n\), \(|S| = |L| = |W| = \lceil \frac{n}{10} \rceil\) and \(|R| = |P| = \lceil \frac{n}{5} \rceil\), where \(n\) represents an integer number. The number of potential size locations \(t \in T\) was set to 3.

Unlike in [3], the number of production centers that can manufacture each product \(|L_p|\), the number of suppliers of each raw material \(|S_r|\) and the number of warehouses that can handle each product \(|W_p|\) are random integer values drawn from a \(U[a,b]\), where \(a\) was set to \(0.25 \times Num\), \(b\) set to \(0.5 \times Num\), and \(Num\) set to \(|S|\), \(|L|\), and \(|W|\), for suppliers, production centers and warehouses, respectively. Once determined in number, they are randomly selected without replacement.

In what concerns production capacities, the parameters of our model are total and per product production capacities, and they are type dependent. Therefore, we had to work out our own methodology to randomly generate these parameters as follows.

Firstly, the per product production usage was randomly drawn from a uniform distribution in the range [1, 10]. Then, taking into account the customer demands and the per product production usage the production capacity required to fulfill all the demand \((\text{TotProd})\) as well as the production capacity to fulfill the per product demand \((\text{TotPerProd})\) were computed. Finally, for each production center of size 1, the total production capacity was drawn from a uniform distribution \(U[a,b]\), where \(a=\text{TotProd}/|L|\) and \(b=\text{TotProd}\).

In an analogous way, the per product production capacity was drawn from a uniform distribution \(U[a,b]\) where \(a=\text{TotPerProd}/|L_p|\) and \(b=\text{TotPerProd}\). For production centers of size 2 and size 3, the total and per product production capacities were set to 95% and 90% of the ones obtained for production centers of size 1.

For the random generation of the storage capacities, the same methodology was used.
The flow magnitude is determined by the number of raw materials from which products are made of, and by the number of customers with a positive demand for each product. These parameters were set to integer numbers drawn from a uniform distribution in the range \([a, b]\) with \(a = 0.5 \times |R|\) and \(b = 0.75 \times |R|\), or \(a = 0.5 \times |C|\), and \(b = 0.75 \times |C|\) for the number of raw materials and for the customers with positive demand, respectively. Once these parameters are defined, the per product customer demands \((d^p_c)\), and the per product usage of raw materials \((a^rp)\) were randomly drawn from a \(U[1,10]\).

Moreover, the raw material availabilities where drawn from a \(U[a,b]\), where \(a\) was set to \(\text{TotRM}' / S^r\) and \(b\) set to \(S \times \text{TotRM}' / S^r\). To compute \(\text{TotRM}'\), which represents the amount of raw material \(r\) necessary to fulfill all the customer demands, the per product usage of raw material and the customers demands were taking into account.

The cost structure is composed by variable costs and by fixed costs. Variable costs include unit purchasing costs, unit transportation costs, unit production costs and unit penalty costs. Exception made for penalty costs, the same methodology was used to randomly generate all the other costs. Firstly, a random number \((\text{rand})\) was drawn from a \(U[1,10]\). Then, the purchasing costs, production costs and transportation costs were randomly drawn from \(U[0.75 \times \text{rand}; 1.25 \times \text{rand}]\).

Penalty costs are acquisition costs for outsourced finished products, and so to make ”home-made” products competitive with the outsourced ones, they were generated considering average per product and per raw material variable costs, average fixed and operational costs for production centers, and average transportation costs.

For production centers and warehouses of size 1, fixed costs were drawn from a \(U[10E+05,10E+06]\) and \(U[10E+04,10E+05]\) whereas operational costs were set to zero. For size 2 and size 3, these costs were set to 95% and to 90% of the ones obtained for size 1, respectively. For fixed transportation costs, we considered two types of instances. Instances in which there are no fixed costs for the modes of transportation, hereafter named as NFC instances, and instances in which there is a fixed cost for each mode of transportation, hereafter named as FC instances. The FC instances differ from the NFC instances on the fixed and unit variable transportation costs. For the FC instances, the unit variable transportation costs are 80% of the ones considered for the NFC instances whereas the fixed transportation costs were set to 20% of the average cost for delivering products.

Finally, for transportation modes capacities we also used our own methodology. Based on our earlier experience with the cement company, we considered three transportation modes: by sea through ships, by land through trains, or by road through trucks. However, some conditions were imposed: cargo ships are available to deliver only a subset of products from a subset of production centers to a subset of warehouses, cargo trains cannot be used to deliver
products to customers, and there is no origin-destination limitation for deliveries made by trucks. Moreover, for truck transportation mode it was assumed that capacity constraints are not binding. For train and ship transportation modes the total and per product capacity between each origin-destination pair was randomly generated as follows. Firstly, for train transportation mode the number of origins and destinations was set to \(\left\lfloor 0.75 \times a \right\rfloor\), with \(a = |S|\), or \(a = |L|\), or \(a = |W|\) depending on what is being generated, whereas for cargo transportation mode this number was set to \(\left\lfloor 0.25 \times a \right\rfloor\) with \(a = |L|\), or \(a = |W|\). Then, they are randomly selected without replacement. Secondly, we took into account only the subset of the selected origins-destinations pairs to randomly generate the total, the per product and the per raw material transportation capacities. These random numbers were drawn from a uniform distribution within a range that varies from the whole required capacity divided by the number of selected origin-destination links to the whole required capacity.

### 4.2 Computational tests

In our experiments we considered three different sizes, \(n = 100, 150, 200\), and two cost structures: transportation modes with fixed costs (FC instances) and without fixed costs (NFC instances). It worths to say that it was impossible to test instances with higher dimensions because CPLEX reported out of memory errors while reading the input data.

In what concerns the \(\alpha_k^t\) parameters, \(k \in K\), we considered two scenarios: \(\alpha_k^t = 0.0\), for all \(k\) and \(t\), and \(\alpha_k^t = 0.2\), for all \(k\) and \(t\). The first scenario allows to simulate situations in which there are no minimum levels of service whereas the second one imposes that once operating production centers and warehouses must fill up 20% of their total production capacities and total storage capacities, respectively. Moreover, the \(\beta\) parameter was set to 0.2, meaning that no more than 20% of the finished products can be outsourced. Therefore, 12 classes of instances were considered. For each class, 5 instances were randomly generated leading to a total of 60 instances.

Computational tests were performed on a Intel Core i3 (3.2 GHz) processor with 4.0 Gb of RAM. The CPLEX 12.3 was used and the time limit for running time was set to 3 hours.

We will start the analysis by evaluating how the suggested valid inequalities are effective, or not, to strengthen the linear programming relaxation of the proposed model. Thus, the minimum, the maximum and the average percent gaps as well as the running time are given in Table 1, for the NFC instances, and in Table 2, for the FC instances. For each instance, the percent gap is calculated as \(\frac{UB^{best} - LB}{UB^{best}}\) where \(UB^{best}\) is the the best known upper bound and \(LB\) is the lower bound obtained by solving the linear programming relaxation. These two tables contain six columns, named as M, M+(29), M+(32), M+(33), M+(38), and M2 that represent model (1)–(28), model (1)–(28) with the additional constraints (29), (32), (33),
For the NFC instances, one can simply run the model (1)–(28) or run the simplified model M1. Therefore, it worths to compare the performance of both models, and so Table 1 contains an extra column, which tabulates the results obtained with the model M1. Note that, we will not give results for models M+(30) and M+(31) since in number they are very similar to model M+(29), and they do not lead to any interesting conclusion.

Finally, note that the running times reported for models M+(38) and M2 do not include the running time for solving neither the model (34)–(37) nor the model (39)–(43). However, they are meaningless: less than 4 seconds. Moreover, solving model (39)–(43) allowed to strengthen all or nearly all the production capacity constraints. Nonetheless, the improvements on gaps provided by model M2 are not significant, namely if compared with model M+(29), as it can be seen in the tabulated results.

Table 1: Minimum, maximum and average percent gaps and running times for the NFC instances with the first and the second scenario.

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<th>n</th>
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<th>M+(32)</th>
<th>M+(33)</th>
<th>M+(38)</th>
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Table 1: Minimum, maximum and average percent gaps and running times for the NFC instances with the first and the second scenario.
Table 1 shows that for both scenarios, and exception made for the second scenario with
$n = 150$, there are no differences between the minimum, maximum and average percent gaps
provided by models M, M+(32), M+(33), M+(38), M1, and M2. For this particular case, the
average percent gap given by the model M is slightly worst than the others. Moreover, it can
be stated that model M+(29) allows to obtain significant improvements when compared with
the other models: the average percent gap decreases 24%, 94% and 97%, for sizes $n = 100$,
$n = 150$ and $n = 200$, scenario 1 and 65%, 92% and 97% for scenario 2, respectively, for sizes
$n = 100$, $n = 150$ and $n = 200$.

In what concerns the running time, it is difficult to establish any relation between the
proposed models since the results depend on the model, on the size, and on the scenario.
However, it worths to say that with the model M+(29) the average running time is, in almost
of the cases, the highest one.

Additionally, it can be seen that the average percent gap decreases with the size whereas
the running time is increasing exponentially, no matter the scenario and the model that
is being considered. Furthermore, the percent average gaps obtained with scenario 1, and
exception made for size $n=100$, are smaller than those obtained with scenario 2.

Another interesting conclusion is that the average results obtained with models $M$ and $M1$
are very similar: the same percent gaps and almost the same running times. Trying to find
out the reason why this occurs, we analyzed the optimal solutions obtained with model $M$,
and in all of them the $Y$ variables are set to 1. Probably, this is done during the preprocessing
phase, and so it becomes indifferent to run the model $M$ or to run the model $M1$.

The percent gaps and the running times obtained with the FC instances, are shown in
Table 2. In this case, the column named as $M1$ does not exist since it makes no sense to
considerer a model in which the $Y$ variables are dropped. The tabulated results allow to state
similar conclusions as the ones established for the NFC instances.

Comparing the results obtained with the NFC instances and the FC instances, it can be
seen that the average percent gaps for $n=200$ are higher for the previous instances than
for the former one, whereas there are no significant differences for the other dimensions.
Moreover, it can be seen that the running times, namely for sizes $n = 150$ and $n = 200$,
are greater for the FC instances than for the NFC instances. These two conclusions can
indicate that the FC instances are harder to solve than the NFC instances, namely for larger
dimensions.

To evaluate the quality of the solution gap provided by CPLEX branch-and-cut, all the
instances were tested and the results are tabulated in Table 3, for the NFC instances, and in
Table 4, for the FC instances. For each of the problem instances, the percent solution gap is
defined as $\frac{UB-LB}{UB}$ where $UB$ and $LB$ are the best upper and lower bounds.

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To start with, it is important to note that some of the models led to out of memory errors, for some of the NFC and FC instances. Nevertheless, for the NFC instances all the models were able to find an upper bound on the optimal value. On the contrary, for the largest size (n = 200) FC instances it was not possible to find always an upper bound on the optimal value, as it can be seen in Table 5: with scenario 1, only with model M+(29) was possible to obtain an upper bound for all the 5 instances tested; with scenario 2, only models M+(29) and model M2 were able to find upper bounds, respectively, 2 out of 5 and 1 out of 5.

To address this problem we had to devise another way to get the missing upper bounds. One way to do it, is to simplify the model by fixing the values of some variables. In our case, this is possible to do since the FC and the NFC instances, and exception made for fixed and variable transportation costs, share the same data. Therefore, we used the best solution obtained with the model M+(29) for the NFC instances to fix the Z and the X variables.
This simplified version of the model M+(29) was then run for the FC instances, and it was possible to obtain an upper bound for each of the 5 instances. As it was previously mentioned, the percent average solution gaps were computed with the best known upper bound. Table 3 shows that the average percent solution gaps are small, ranging from 0.00% to 7.50%, for scenario 1 and from 0.00% to 8.27%, for scenario 2. Nevertheless, the model M+(29) outperforms all the other models. For sizes \( n = 100 \) and \( n = 150 \), in which all the models give an average solution gap of 0%, model M+(29) gives it much faster. For size \( n = 200 \), there are significant differences. The percent solution gap provided by the model M+(29) and by all the other models are 0.01% and 0.03% against average gaps that ranges over 2.91 – 7.50% and 5.00 – 8.27% for scenario 1 and 2, respectively. Furthermore, the average running times are 729.26 s and 2973.59 s against running times that ranges over 9247.43 – 9746.88 s and 9521.86 – 10693.41 s for scenario 1 and 2, respectively.

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<th>Scenario</th>
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<th>M</th>
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<th>M+(32)</th>
<th>M+(33)</th>
<th>M+(38)</th>
<th>M1</th>
<th>M2</th>
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<td>43.66</td>
<td>289.45</td>
<td>122.92</td>
<td>100.75</td>
<td>100.75</td>
<td>100.75</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>Avg 89.58</td>
<td>25.12</td>
<td>115.77</td>
<td>58.33</td>
<td>55.27</td>
<td>55.27</td>
<td>55.27</td>
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</tr>
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</table>

Table 3: Average, minimum and maximum percent solution gaps for the NFC instances with the first and the second scenario.
### Table 4: Average, minimum and maximum percent solution gaps for the FC instances with the first and second scenario

<table>
<thead>
<tr>
<th>Scenario</th>
<th>n</th>
<th>M</th>
<th>M+(29)</th>
<th>M+(32)</th>
<th>M+(33)</th>
<th>M+(38)</th>
<th>M2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scenario 1</strong></td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dev (%)</td>
<td>Min</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Avg</td>
<td>0.00</td>
<td>261.77</td>
<td>105.04</td>
<td>530.98</td>
<td>527.00</td>
<td>274.61</td>
</tr>
<tr>
<td>Time (s)</td>
<td>Max</td>
<td>10800.00</td>
<td>7859.72</td>
<td>10800.00</td>
<td>10800.00</td>
<td>5182.98</td>
<td>6324.76</td>
</tr>
<tr>
<td></td>
<td>Avg</td>
<td>3790.20</td>
<td>2657.16</td>
<td>7141.65</td>
<td>4753.44</td>
<td>1395.13</td>
<td>1614.99</td>
</tr>
<tr>
<td><strong>Scenario 2</strong></td>
<td>150</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dev (%)</td>
<td>Min</td>
<td>0.00</td>
<td>0.00</td>
<td>1.27</td>
<td>1.39</td>
<td>0.00</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>7.35</td>
<td>0.00</td>
<td>9.84</td>
<td>12.98</td>
<td>6.14</td>
<td>6.75</td>
</tr>
<tr>
<td></td>
<td>Avg</td>
<td>3.83</td>
<td>0.00</td>
<td>5.14</td>
<td>6.83</td>
<td>3.30</td>
<td>3.77</td>
</tr>
<tr>
<td>Time (s)</td>
<td>Max</td>
<td>10800.00</td>
<td>432.70</td>
<td>10800.00</td>
<td>8721.78</td>
<td>10283.21</td>
<td>10800.00</td>
</tr>
<tr>
<td></td>
<td>Avg</td>
<td>10800.00</td>
<td>4752.40</td>
<td>7529.21</td>
<td>6152.39</td>
<td>4148.68</td>
<td>4522.55</td>
</tr>
<tr>
<td><strong>Scenario 2</strong></td>
<td>200</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dev (%)</td>
<td>Min</td>
<td>5.23</td>
<td>0.07</td>
<td>5.24</td>
<td>5.23</td>
<td>0.39</td>
<td>8.06</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>18.53</td>
<td>0.64</td>
<td>13.74</td>
<td>18.53</td>
<td>13.74</td>
<td>13.73</td>
</tr>
<tr>
<td></td>
<td>Avg</td>
<td>10.96</td>
<td>0.33</td>
<td>9.39</td>
<td>10.96</td>
<td>9.27</td>
<td>10.87</td>
</tr>
<tr>
<td>Time (s)</td>
<td>Max</td>
<td>10800.00</td>
<td>452.89</td>
<td>2545.66</td>
<td>3387.44</td>
<td>2580.88</td>
<td>2991.97</td>
</tr>
<tr>
<td></td>
<td>Avg</td>
<td>10800.00</td>
<td>944.73</td>
<td>6739.93</td>
<td>4084.30</td>
<td>590.22</td>
<td>661.33</td>
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</table>

### Table 5: Number of upper bounds

<table>
<thead>
<tr>
<th>Model</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>M+(29)</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>M+(32)</td>
<td>0</td>
<td>0</td>
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<tr>
<td>M+(33)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>M+(38)</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>M2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

It can also be seen that for a given value of $n$, the average percent solution gap and the average running time are stable over the scenarios, even if for scenario 2 they tend to be higher. Moreover, across the problem size ($n$) and for both scenarios, the average running time is increasing exponentially. Unlike it happens with the linear programming relaxation, there are now differences between the models, and the worst gaps are provided by model M+(33).
Table 4, which tabulates the results for the FC instances, allows us to state the same conclusion that was done for the NFC instances: model M+(29) gives much better percent average solution gaps and much faster than the other models. Comparing the results obtained for both types of instances, one can see that, exception made for size $n = 100$, the percent average solution gaps are worse for the FC instances, e.g. for size $n = 150$ in which gaps range over $0.00 - 6.83\%$ against gaps that range over $0.00 - 0.01\%$. In what concerns running times, a not fair comparison can be made because many out of memory errors were obtained.

Thus, it becomes now clear that FC instances seem to be harder to solve to optimality than the NFC ones. Moreover, across the problem size ($n$), the scenarios, and the type of instances the model M+(29) is the only one that remains stable.

5 Conclusions and future research

This research explores the use of outsourcing within the context of logistic network. The contributions of this research lies in the development of a modeling framework for an intermodal integrated supply chain network design with outsourcing, and a set of valid inequalities to the proposed mixed integer linear programming model.

The modeling framework incorporates different types of shipments (directly from production centers, or via warehouses), different modes of transportation, minimum levels of service and outsourcing strategies.

The proposed model as well as the valid inequalities have been tested on a set of randomly generated instances of varying dimensions and different settings with CPLEX 12.3.

The obtained results in terms of the quality of the percent solution gap and the running times show that one set of the proposed valid inequalities led to significant improvements: it was possible to obtain average solution gaps that range over $0.00 - 0.78\%$, and much faster than with the other models, namely the main one where no valid inequalities were considered.

This study also found that instances in which a fixed cost for modes of transportation is considered are harder to solve to optimality. Nevertheless, with the former set of valid inequalities the average gaps remain stable across the problem size and the different settings.

The model proposed in this paper can be extended by considering other relevant issues. Our future research will be towards the incorporation of these issues in the model, namely time dependency by allowing a multi-period planning horizon, data uncertainty, e.g. customer demands not deterministic, and single source.

A Notation

Table 6 summarizes the notation for parameters.
Notation

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>Set of customers</td>
</tr>
<tr>
<td>$L$</td>
<td>Set of potential plant locations</td>
</tr>
<tr>
<td>$P$</td>
<td>Set of products</td>
</tr>
<tr>
<td>$R$</td>
<td>Set of raw materials</td>
</tr>
<tr>
<td>$S$</td>
<td>Set of potential suppliers</td>
</tr>
<tr>
<td>$T$</td>
<td>Set of types that are available</td>
</tr>
<tr>
<td>$W$</td>
<td>Set of potential warehouse locations</td>
</tr>
<tr>
<td>$D = W \cup L \cup C$</td>
<td>Set of destinations</td>
</tr>
<tr>
<td>$I = R \cup P$</td>
<td>Set of items</td>
</tr>
<tr>
<td>$K = L \cup W$</td>
<td>Set of facilities</td>
</tr>
<tr>
<td>$O = S \cup L \cup W$</td>
<td>Set of origins</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Set of customers that require product $p$</td>
</tr>
<tr>
<td>$L_p$</td>
<td>Set of potential production centers locations for making product $p$</td>
</tr>
<tr>
<td>$M_{od}$</td>
<td>Set of transportation modes between origin-destination $(o,d)$</td>
</tr>
<tr>
<td>$M_{od}^i$</td>
<td>Set of transportation modes between origin-destination $(o,d)$ for item $i$</td>
</tr>
<tr>
<td>$O^i$</td>
<td>Set of potential origins for item $i$</td>
</tr>
<tr>
<td>$P_r$</td>
<td>Set of products that require raw material $r$</td>
</tr>
<tr>
<td>$S^r$</td>
<td>Set of potential suppliers providing raw material $r$</td>
</tr>
<tr>
<td>$W_p$</td>
<td>Set of potential warehouse locations for storing product $p$</td>
</tr>
</tbody>
</table>

$\alpha^p$ | Usage of raw material $r$ in product $p$ |
$AP_{ot}$ | Production capacity at origin $o$ of type $t$ |
$AP_{ot}^p$ | Production capacity for product $p$ at origin $o$ of type $t$ |
$ARM_o$ | Availability of raw material $r$ at supplier $s$ |
$AS_t^i$ | Storage capacity at origin $o$ of type $t$ |
$ASC_t^i$ | Storage capacity for product $p$ at origin $o$ of type $t$ |
$CC_{rim}$ | Capacity required per unit of item $i$ in transportation mode $m$ |
$CM_{od}^m$ | Capacity of transportation mode $m$ origin-destination $(o,d)$ |
$CM_{od}^m$ | Capacity of transportation mode $m$ between origin-destination $(o,d)$ for item $i$ |
$CPC_p$ | Capacity required to produce one unit of product $p$ |
$d^c$ | Demand of customer $c$ for product $p$ |
$dist_{oad}$ | Distance between origin $o$ and destination $d$ |
$STC_p$ | Storage space required by product $p$ |
$f_{1k}^t$ | Opening cost for facility $k$ of type $t$ |
$f_{2od}^m$ | Fixed cost of using transportation mode $m$ from origin $o$ to destination $d$ |
$f_{3k}^t$ | Fixed cost for operating facility $k$ of type $t$ |
$c_{1s}^r$ | Unit cost of raw material $r$ at supplier $s$ |
$c_{2m}^i$ | Unit transportation cost for item $i$ using transportation mode $m$ |
$c_{3l}^p$ | Unit cost of producing product $p$ at plant $l$ |
$Pen_p$ | Unit cost of supplying product $p$ from an external supplier |

Table 6: Notation for parameters

References


