A primal decomposition scheme for a dynamic capacitated phase-in/phase-out location problem

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Abstract

We consider a dynamic capacitated facility location problem. Given a set of customers with known demands for a single product, a set of facilities operating at the beginning of the planning horizon, and a set of potential sites to locate new facilities, the objective is to find the location-allocation plan that minimizes the overall costs and satisfies the customer demands without violating the finite capacities of the operating facilities. The problem is modelled as a mixed integer linear program with binary variables associated with location decisions, that is, phase-in (phase-out) of new (existing) facilities, and continuous variables representing the distribution flows from the facilities to the customers. These two different types of decisions lead to a natural separation of the variables, thereby making the problem an attractive candidate for applying a decomposition technique. We propose a method based on primal Benders decomposition, and improve its performance not only by introducing valid inequalities that tighten the lower bound of the linear relaxation but also by developing a heuristic approach that strengthens the usual Benders cuts. For randomly generated problem instances, the computational results show that the new Benders algorithm clearly outperforms standard mathematical programming software, thus making the method attractive for decision-makers who can use it as a tool to redesign their logistics networks and evaluate the impact of alternative network configurations.

Keywords: dynamic facility location, Benders decomposition.

1 Introduction

Dynamic facility location problems (DFLP) deal with planning the location and/or size of facilities over a given time horizon during which changes in customer demands and cost structures are expected to occur. Such problems arise in a wide variety of important domains, including production-distribution system design (see e.g. Canel *et al.* [4]), supply chain planning (see e.g. Arntzen *et al.* [2]), telecommunications network design (see e.g. Chardaire *et al.* [5]), and public facility location (see e.g. Antunes and Peeters [1] for an application to school network planning).

In general terms, the problem addressed by this paper can be described as follows: given a set of customers whose locations and demands are assumed to be known for each period of the planning horizon, a set of existing facilities already operating at the beginning of the time horizon, and a set of potential sites where new facilities can be established, it is required to determine which facilities should be used during each period so that the overall costs, including facility location and operation costs as well as distribution costs, are minimized subject to satisfying customer demands without exceeding the capacity of each operating facility in every period. Observe that the location and time-phasing decisions concern not only the phase-in of new facilities, as in classical location problems, but also the phase-out of initially existing facilities. This allows to model situations in which a set of facilities is already in place but is inappropriate to cope with changing market conditions. As a result, the network structure needs to be reorganized or redesigned. Triggered by the globalization of the economy and fierce competition in the marketplace, companies face the growing challenge to constantly evaluate and reconfigure the structure of their logistics networks as well as the strategies for providing a desired customer service at the lowest possible cost. Hence, changing economic conditions compel companies to include the effect of the future time dimension in their location analysis, thereby justifying the need for dynamic facility location models.

Dynamic facility location has been a field of recurring interest as demonstrated by the recent surveys by Klose and Drexl [18], Owen and Daskin [24], and ReVelle and Eiselt [25]. The problem to be addressed in this paper generalizes the dynamic uncapacitated facility location problem (DUFLP) studied by Van Roy and Erlenkotter [30]. The authors developed a branch-and-bound procedure with lower bounds obtained with a heuristic dual ascent method, and upper bounds constructed with the dual solutions and the complementary slackness conditions. The phase-in version of the DUFLP was examined by Frantzeskakis and Watson-Gandy [11] who proposed a branch-and-bound method. Lower bounds were determined by solving in each node of the search tree a subproblem with dynamic programming applied to a reduced state space. Hormozi and Khumawala [15] also developed an exact algorithm for the DUFLP combining mixed integer and dynamic programming methods. Single-period solutions are first obtained for each period and the optimal sequence for the complete planning horizon is later determined by applying dynamic programming. This technique was later extended by Canel *et al.* [4] to the DFLP with multiple commodities and capacitated facilities. The proposed algorithm was applied to a single instance of reduced size and reflects the main drawback of most exact solution approaches, namely their prohibitive time and memory requirements which do not make them attractive to solve realistically sized problems. An exception to this seems to be the method developed by Dogan and Goetschalckx [10] for the configuration of the production-distribution system of a real-life manufacturer of cardboard packages. The authors applied Benders decomposition to a network design problem with multiple commodities and different types of facilities (production plants and warehouses) with limited capacities. The goal is to find a facility configuration that is robust enough to cope with changes that are expected to occur in all parameters during the planning horizon. The location decisions are, however, static in the sense that they are made at the beginning of the time horizon. For the selected facility configuration, a production and distribution plan is then determined for each planning period.

The limited success of exact methods has prompted the development of heuristic procedures, many of which have performed very well in their search for near-optimal solutions. Among the various strategies available in the literature, Chardaire *et al.* [5] find feasible solutions for the DUFLP by simulated annealing and generate lower bounds by Lagrangean relaxation. The Lagrangean relaxation technique was also used by Shulman [28] to solve a special version of the capacitated DFLP. The author considers that a new facility can be established in each potential site with various modules of different sizes. This problem arises, for example, in the design and expansion of telecommunication networks where different types of concentrators are to be installed in a node, and the size of each node can be increased during the planning horizon through the installation of additional capacity. Shulman [28] also used Lagrangean relaxation to convert infeasible solutions of the relaxed problem into feasible solutions. Modular capacities were also considered by Antunes and Peeters [1] for facility location, expansion and reduction in the context of school network planning. The problem was solved by simulated annealing. Hinojosa *et al.* [13] applied Lagrangean relaxation to a problem combining dynamic aspects with multi-stage facility location in a multi-commodity distribution network. Velten [31] later extended the proposed model to incorporate inventory decisions. Recently, Dias *et al.* [9] proposed a primal-dual heuristic for a capacitated version of the DFLP allowing opening and closing facilities more than once during the planning horizon. Finally, Saldanha da Gama [26] developed a heuristic approach for the problem to be examined in this paper. It starts by generating lower bounds with a dual heuristic procedure. Feasible primal solutions are then obtained using the dual solutions and improving them further by local search. An extensive computational study has indicated that the quality of the final feasible solutions is influenced by the quality of the lower bounds previously obtained. Although on average good solutions could be found, in some extreme cases the quality of the final solutions deteriorated as a result of having derived them from poor lower bounds. The contribution of this paper is to overcome these difficulties by proposing an efficient alternative method. Since the structure of the capacitated DFLP is well suited for a primal decomposition approach such as Benders decomposition, as will be shown, we will focus on applying this technique to our problem. To the best of our knowledge, this is the first research effort to examine the benefits of solving a dynamic phase-in/phase-out facility location problem by Benders decomposition.

The remainder of the paper is organized as follows. Section 2 presents a mathematical formulation of the problem as a mixed integer linear program while Section 3 describes the solution methodology. The results of an extensive computational study are reported in Section 4. Finally, conclusions and directions for further research are given in Section 5.

2 Problem formulation

Given a set of facilities already in place, a set of possible sites for establishing new facilities and a set of customers with known demands throughout a pre-specified time horizon, the dynamic capacitated facility location problem ($DCFLP$) consists in determining the location plan that describes *when* and *where* the phase-in of new facilities and the phase-out of existing facilities should take place during the planning horizon. In particular, the location-allocation plan that minimizes the overall costs for meeting the customer demands in each period without exceeding the maximum capacities of the facilities in operation is sought. The following notation is used in formulating the $DCFLP$.

T : index set of periods in the planning horizon with $n = \mid T \mid$

I^c : index set of locations at which initially existing facilities may be closed

I^o : index set of locations at which new facilities may be opened

I : index set of all facility locations, $I = I^c \cup I^o$, $I^c \cap I^o = \emptyset$

Q_i : capacity of facility $i \in I$

J : index set of customer locations

The following parameters are included in the model:

d_{jt} : demand of customer $j \in J$ in period $t \in T$

c_{ijt} : variable cost of serving all demand of customer $j \in J$ in period $t \in T$ by facility $i \in I$

F_{it} : fixed cost for opening or closing facility $i \in I$ in period $t \in T$

For each existing facility $i \in I^c$, F_{it} includes not only the fixed charge for closing the facility at the end of period t but also the overall costs resulting from having kept the facility in operation from period 1 through period t. Analogously, for each new location $i \in I^o$, F_{it} comprises the fixed cost for opening facility i at the beginning of period t as well as all future operating costs until the end of period n.

The following decision variables are defined:

x_{ijt} : fraction of demand of customer $j \in J$ served by facility $i \in I$ in period $t \in T$

$$z_{it} = \begin{cases} 1 & \text{if the configuration of facility } i \in I \text{ changes in period } t \in T \\ 0 & \text{otherwise} \end{cases}$$

Since any existing facility $i \in I^c$ is initially operating, a configuration change amounts to closing the facility at the end of a given period. It is assumed that it is not possible to close such a facility at the end of the last period n since the impact of this decision would only be noticeable after the planning horizon. Hence, $z_{in} = 0$ for every $i \in I^c$. Concerning the setup of new facilities, since any site $i \in I^o$ is not used at the beginning of the location study, changing its configuration corresponds to opening a new facility there at the start of a given period. The establishment of new facilities is typically a time-consuming project due to preparation activities such as facility construction, equipment supply and employee training. Therefore, it is assumed that new facilities can only start operating at the beginning of the second period or later, and so $z_{i1} = 0$ for every $i \in I^o$.

The $DCFLP$ is formulated as a mixed integer linear program as follows.

$$\text{Min} \quad \sum_{i\in I}\sum_{j\in J}\sum_{t\in T} c_{ijt}\, x_{ijt} + \sum_{i\in I^c}\left[\sum_{t=1}^{n-1} F_{it} z_{it} + F_{in}\Big(1 - \sum_{\tau=1}^{n-1} z_{i\tau}\Big)\right] + \sum_{i\in I^o}\sum_{t=2}^{n} F_{it} z_{it} \qquad (1)$$

subject to

$$\sum_{i\in I} x_{ijt} = 1 \qquad j\in J,\ t\in T \qquad (2)$$

$$\sum_{j\in J} d_{jt}\, x_{ijt} \le Q_i \left(1 - \sum_{\tau=1}^{t-1} z_{i\tau}\right) \qquad i\in I^c,\ t\in T \qquad (3)$$

$$\sum_{j\in J} d_{jt}\, x_{ijt} \le Q_i \sum_{\tau=2}^{t} z_{i\tau} \qquad i\in I^o,\ t\in T \qquad (4)$$

$$\sum_{t=1}^{n-1} z_{it} \le 1 \qquad i\in I^c \qquad (5)$$

$$\sum_{t=2}^{n} z_{it} \le 1 \qquad i\in I^o \qquad (6)$$

$$x_{ijt} \ge 0 \qquad i\in I,\ j\in J,\ t\in T \qquad (7)$$

$$z_{it} \in \{0,1\} \qquad i\in I,\ t\in T \qquad (8)$$

The objective function (1) minimizes the sum of all fixed and variable costs. Constraints (2) require the demand at each customer location to be satisfied throughout the planning horizon. Constraints (3) and (4) ensure that the total demand served by each facility in each period does not exceed the available capacity. Inequalities (5) and (6) allow the configuration of each facility to change at most once during the planning horizon. This means that once an existing (new) facility is closed (opened) it will not be later re-opened (closed). Finally, constraints (7) and (8) impose non-negativity and integrality conditions.

The *DCFLP* is $\mathcal{NP}$-hard since for $n = 1$ and infinite capacities the problem reduces to the classic uncapacitated facility location problem which is known to be $\mathcal{NP}$-hard, Jacobsen [16]. Facility location models are frequently enhanced with valid inequalities. The most common inequalities are those stating that customer demands can only be satisfied from open facilities. In the static case, the inclusion of these inequalities in both the uncapacitated and capacitated variants leads to *strong* formulations, meaning that the lower bounds of the corresponding LP-relaxations become much tighter in the presence of these constraints (see Klose and Drexl [18], Leung and Magnanti [21]). Their extension to the $DCFLP$ is straightforward:

$$x_{ijt} \leq 1 - \sum_{\tau=1}^{t-1} z_{i\tau} \qquad i \in I^c,\ j \in J,\ t \in T \tag{9}$$

$$x_{ijt} \leq \sum_{\tau=2}^{t} z_{i\tau} \qquad i \in I^o,\ j \in J,\ t \in T \tag{10}$$

Observe that the above constraints are redundant since the values of the location variables z_{it} are restricted to 0 or 1 by (8).

In static capacitated facility location problems the inclusion of a so-called *total demand constraint* also seems to very useful while solving these problems by a decomposition scheme. This condition ensures that the available capacity at the operating facilities is enough to meet the total customer demands. Its extension to the $DCFLP$ is also straightforward.

$$\sum_{i \in I^c} Q_i \left(1 - \sum_{\tau=1}^{t-1} z_{i\tau}\right) + \sum_{i \in I^o} Q_i \left(\sum_{\tau=2}^{t} z_{i\tau}\right) \geq \sum_{j \in J} d_{jt} \qquad t \in T \tag{11}$$

As will be shown in the next section, primal decomposition can highly benefit from including constraints (9)–(11) in the $DCFLP$.

3 Benders decomposition

Discrete facility location problems are attractive candidates for decomposition techniques since they contain two types of inherently different decisions: on the one hand, the yes/no-decision where to locate facilities (variables z_{it}), and on the other hand how best to allocate customer demands to the selected facilities (variables x_{ijt}). Once the discrete-choice site selection has been made, the resulting linear program becomes much simpler to solve. Benders decomposition is a well-known technique, originally introduced by Benders [3], which exploits the special structure of mixed integer linear problems. Decoupling the binary decision variables from the continuous variables in the $DCFLP$ leads to a master problem which fixes the startup and shutdown schedules of the facilities. The continuous variables are placed in a subproblem whose solution sends marginal information to the master problem regarding the "goodness" of the proposed startup and shutdown schedule. This information is provided by the dual variables of the subproblem which are used to create in each iteration a so-called *Benders cut* that is added to the master problem.

The solution of the latter problem suggests a new startup and shutdown schedule. The procedure continues until convergence to the optimal solution is attained (see Lasdon [19] and Magnanti and Wong [22] for a detailed description of the method).

Benders decomposition has been successfully applied to static facility location problems, see e.g., Geoffrion and Graves [12], Holmberg [14], Klose [17], Lee [20] and Wentges [32]. Recently, Cordeau *et al.* [6] proposed the technique to solve a comprehensive problem arising in the design of logistics networks. Van Roy [29] combined Benders decomposition with Lagrangian relaxation to exploit simultaneously the primal and dual structures of the static capacitated facility location problem. The resulting method is known as cross decomposition. To the best of our knowledge, Benders decomposition has never been applied to a phase-in/phase-out DFLP before. Therefore, this paper presents a first contribution in that direction and highlights the benefits of the technique.

3.1 Decomposition scheme

Consider formulation (1)-(8) with the valid inequalities (9)-(11). For a given feasible facility configuration (i.e., a feasible $\mathbf{z}$ vector), the resulting subproblem reduces to an allocation problem involving only the distribution variables $\mathbf{x}$. Let $SP_{\mathbf{z}}$ denote this subproblem for a given realization of $\mathbf{z}$. The $DCFLP$ can be easily stated in a compact form involving the subproblem $SP_{\mathbf{z}}$:

$$\underset{\mathbf{z}\in\mathbf{Z}}{\text{Min}} \quad \underbrace{\underset{\mathbf{x}\geq\mathbf{0}}{\text{Min}} \Big\{ (1) \text{ subject to } (2),\ (3),\ (4),\ (7),\ (9),\ (10) \Big\}}_{SP_{\mathbf{z}}} \tag{12}$$

with

$$\mathbf{Z} = \{z_{it},\ i \in I, t \in T : (5),\ (6),\ (8),\ (11)\ \}$$

The set $\mathbf{Z}$ includes all feasible facility configurations of the $DCFLP$. Multiplying constraints (3), (4), (9) and (10) by (-1), and associating dual variables $\boldsymbol{\lambda} = \{\lambda_{jt} : j \in J, t \in T\}$ to (2), $\boldsymbol{\nu} = \{\nu_{it} : i \in I, t \in T\}$ to (3) and (4), and $\boldsymbol{\pi} = \{\pi_{ijt} : i \in I, j \in J, t \in T\}$ to (9) and (10), the dual problem of $SP_{\mathbf{z}}$ - denoted by $DSP_{\mathbf{z}}$ - can be formulated as:
with

$$K = \sum_{i\in I^c}\left[\sum_{t=1}^{n-1} F_{it}\, z_{it} + F_{in}\left(1 - \sum_{\tau=1}^{n-1} z_{i\tau}\right)\right] + \sum_{i\in I^o}\sum_{t=2}^{n} F_{it}\, z_{it}$$

$$\text{Max} \quad K + \sum_{j\in J}\sum_{t\in T}\lambda_{jt} - \sum_{i\in I^c}\sum_{t\in T}\nu_{it}\,Q_i\left(1-\sum_{\tau=1}^{t-1}z_{i\tau}\right) - \sum_{i\in I^o}\sum_{t\in T}\nu_{it}\,Q_i\left(\sum_{\tau=2}^{t}z_{i\tau}\right)$$

$$-\sum_{i\in I^c}\sum_{j\in J}\sum_{t\in T}\pi_{ijt}\left(1-\sum_{\tau=1}^{t-1}z_{i\tau}\right) - \sum_{i\in I^o}\sum_{j\in J}\sum_{t\in T}\pi_{ijt}\left(\sum_{\tau=2}^{t}z_{i\tau}\right) \tag{13}$$

subject to

$$\lambda_{jt} - d_{jt}\,\nu_{it} - \pi_{ijt} \le c_{ijt} \qquad i \in I,\ j \in J,\ t \in T \tag{14}$$

$$\nu_{it} \ge 0 \qquad i \in I,\ t \in T \tag{15}$$

$$\pi_{ijt} \ge 0 \qquad i \in I,\ j \in J,\ t \in T \tag{16}$$

The polyhedron defined by constraints (14)-(16) does not depend on the binary variables $\mathbf{z}$ which only appear in the objective function of $DSP_{\mathbf{z}}$. Observe that the primal subproblem $SP_{\mathbf{z}}$ can be decomposed into n transportation problems, one for each period, which often have a high level of degeneracy. This may result in multiple optimal solutions for the above dual subproblem $DSP_{\mathbf{z}}$. This characteristic will be later explored to improve the performance of the proposed method (see Section 3.2).

Denoting the set of extreme points of (14)-(16) by $\Lambda\Pi V$, problem (12) can be rewritten as a minimax problem as follows:

$$\underset{\mathbf{z}\,\in\,\mathbf{Z}}{\text{Min}} \ \left\{ \text{Max (13)} \ : \ (\boldsymbol{\lambda},\ \boldsymbol{\pi},\ \boldsymbol{\nu}) \in \Lambda\Pi V \right\} \tag{17}$$

Since $\mathbf{z} \in \mathbf{Z}$, there is enough capacity at the operating facilities to serve all customer demands. Hence, there exists a vector $\overline{\mathbf{x}}$ satisfying all constraints of the subproblem $SP_{\mathbf{z}}$. Moreover, given that each $x_{ijt} \in [0,1]$ and all costs in the objective function of $SP_{\mathbf{z}}$ are non-negative, it follows that the subproblem is always feasible and bounded. Consequently, the set $\Lambda\Pi V$ is always bounded and as a result, it is not necessary to consider the extreme rays of the set defined by (14)-(16).

The linearization of problem (17) leads to the so-called *Benders master problem*:

$$\text{Min} \qquad \rho \tag{18}$$

subject to

$$\rho \ge \text{ (13)} \qquad (\boldsymbol{\lambda},\ \boldsymbol{\pi},\ \boldsymbol{\nu}) \in \Lambda\Pi V \tag{19}$$

$$\mathbf{z} \in \mathbf{Z} \tag{20}$$

where inequalities (19) are known as *Benders cuts* or optimality cuts. Since even in a problem of moderate size the cardinality of $\Lambda\Pi V$ is usually very high, the number of Benders cuts is usually huge. However, not all of them will be binding at an optimal solution. In general, optimality can be obtained by solving a relaxed master problem, that is, by solving the above problem (which is equivalent to the $DCFLP$) with only few constraints of type (19) (see Magnanti and Wong [23] and Van Roy [29]). Starting with only a few (or no) extreme points, the Benders method iterates between a relaxed master problem and the dual subproblem. In each iteration, a relaxed Benders master problem (18)-(20) is solved with the set of cuts available at that iteration. The solution obtained corresponds to a feasible facility configuration $\mathbf{z} \in \mathbf{Z}$, that is, a startup and shutdown schedule for the facilities throughout the planning horizon. Moreover, it provides a lower bound on the optimal solution value of the original problem $DCFLP$. This schedule is used in the dual subproblem $DSP_{\mathbf{z}}$ to obtain a distribution plan for servicing the customer demands. By solving the $DSP_{\mathbf{z}}$, an extreme point of $\Lambda\Pi V$ can be identified which leads to a new Benders cut of the form (19) that is added to the relaxed master problem. The algorithm then proceeds to a new iteration. Observe that an upper bound on the optimal solution value of the $DCFLP$ is available in every iteration (the optimal value of $DSP_{\mathbf{z}}$ is equal to the optimal value of $SP_{\mathbf{z}}$). The process continues until the lower and upper bounds are sufficiently close.

Different relaxed master problems may have the same optimal solution. If this happens, a loop is created in the algorithm. To avoid this, in each iteration an improvement cut is enforced in the relaxed master problem stating that the optimal value of the current relaxed master problem must be strictly greater than the optimal value of the previous relaxed master problem.

The Benders algorithm starts with a feasible facility configuration. A trivial feasible solution for the $DCFLP$ corresponds to setting $z_{it} = 0$, for $i \in I^c, t \in T$, $z_{i2} = 1$ for $i \in I^o$, and $z_{it} = 0$ for $i \in I^o, t \in T \setminus \{2\}$. In other words, all initially existing facilities remain in operation throughout the planning horizon whereas a new facility is opened in every candidate site at the beginning of the second period. Other possibilities for generating initial feasible solutions were also investigated, namely, by solving the relaxed master problem without cuts and using a heuristic procedure especially developed for the $DCFLP$. However, an extensive computational study has shown that these alternative procedures were less effective and required larger computation time (see Saldanha da Gama and Silva [27] for further details).

Benders decomposition is widely known for its high computational effort, which arises mainly from the difficulty in solving the relaxed master problem in each iteration. This is due to the fact that not only a mixed integer linear program has to be solved in each iteration but also the size of the problems gradually increases through adding a new constraint (Benders cut) per iteration. Therefore, a key factor in improving the efficiency of the method concerns the number of Benders cuts needed to reach optimality. Clearly, the greater the number of cuts, the greater the number of relaxed master problems that must be solved. To overcome this difficulty, improved cuts can be derived that make use of the multiplicity of optimal solutions of the dual subproblem $DSP_{\mathbf{z}}$. Observe that having multiple dual optimal solutions leads to a number of valid alternative Benders cuts. However, one cut may be dominated by another in the neighborhood of the optimal solution. This characteristic was explored by Magnanti and Wong [23] and Van Roy [29] for static facility location problems. In the next section, strategies for finding enhanced cuts, that is, cuts that lead to better lower bounds and expedite convergence of the Benders algorithm are introduced.

3.2 Strategies for strengthening Benders cuts

Consider the formulation of the $DCFLP$ with the valid inequalities (9)-(11). The *usual* Benders cut has the form (19) which is equivalent to

$$\begin{aligned}\rho \;\geq\; & K + \sum_{j\in J}\sum_{t\in T}\lambda_{jt} - \sum_{i\in I^c}\sum_{t\in T}\left(\nu_{it}\,Q_i + \sum_{j\in J}\pi_{ijt}\right)\left(1 - \sum_{\tau=1}^{t-1} z_{i\tau}\right) - \\ & \sum_{i\in I^o}\sum_{t\in T}\left(\nu_{it}\,Q_i + \sum_{j\in J}\pi_{ijt}\right)\left(\sum_{\tau=2}^{t} z_{i\tau}\right) \qquad (21)\end{aligned}$$

The above inequality can be strengthened by increasing its right-hand side. The latter is affected by the values of the dual variables $\boldsymbol{\lambda}$, $\boldsymbol{\pi}$ and $\boldsymbol{\nu}$ of the subproblem $DSP_{\mathbf{z}}$, which in turn are associated with some feasible facility configuration $\mathbf{z} \in \mathbf{Z}$.

If $1 - \sum_{\tau=1}^{t-1} z_{i\tau} = 0$ for a certain pair (i,t), $i \in I^c$ and $t \in T$ (which means that $z_{i\tau} = 1$ for some $\tau = 1,\ldots,t-1$), then the corresponding coefficient $\nu_{it}\,Q_i + \sum_{j\in J}\pi_{ijt}$ may be modified while maintaining the dual feasibility. In this case, $1 - \sum_{\tau=1}^{t-1} z_{i\tau}$ is still equal to 0 if we replace $t-1$ by $t, t+1, \ldots, n-1$. Therefore, not only the values of the dual variables ν_{it} and π_{ijt}, $j \in J$ can be altered but also those of the variables $\nu_{i\tau}$ and $\pi_{ij\tau}$ for every $\tau = t+1, \cdots, n$ and $j \in J$. Similarly, if $\sum_{\tau=2}^{t} z_{i\tau} = 0$ for a given pair

(i,t), $i \in I^o$ and $t \in T$, then the corresponding coefficient in (21) can be modified. Note that if $z_{it} = 1$ for some pair (i,t), then the configuration of the facility cannot change in any other period. In particular, the relation $z_{i2} + z_{i3} + \ldots + z_{it-1} = 0$ holds and as a result, the dual variables $\nu_{i\tau}$ and $\pi_{ij\tau}$ can be changed for every $\tau = 2, \ldots, t-1$ and $j \in J$. Finally, if $z_{it} = 0$ for every $i \in I^o$ and $t \in T$ this indicates that all dual variables may have their values modified. Summarizing, to determine the best dual values associated with the corresponding coefficients in (21), the following linear program is solved for every pair (i,t) such that the corresponding coefficient can be changed:

$$\text{Min} \quad \nu_{it}\, Q_i \; + \sum_{j \in J} \pi_{ijt} \tag{22}$$

subject to

$$\lambda_{jt} - d_{jt}\, \nu_{it} - \pi_{ijt} \leq c_{ijt} \qquad j \in J \tag{23}$$

$$\nu_{it} \geq 0$$

$$\pi_{ijt} \geq 0 \qquad j \in J$$

The solution of each problem guarantees that the new values of the dual variables are feasible for $DSP_{\mathbf{z}}$. Knowing that the $\boldsymbol{\pi}$ variables are non-negative, that they must satisfy inequalities (23) and that their values are to be minimized according to (22), it follows that each of the above problems can be rewritten in a condensed form denoted by CP_{it}:

$$\underset{\nu_{it} \geq 0}{\text{Min}} \quad \nu_{it}\, Q_i \; + \sum_{j \in J} \max\{0, \lambda_{jt} - d_{jt}\, \nu_{it} - c_{ijt}\} \tag{24}$$

Solving CP_{it} to optimality for each pair (i,t) may be time consuming when the number of pairs is very large. This was confirmed by our computational study when using standard mathematical programming software, namely ILOG CPLEX 9.0 [8]. Therefore, a heuristic procedure is proposed to find good solutions of CP_{it} with reduced computational effort. The procedure starts with the optimal solution of the dual of subproblem $SP_{\mathbf{z}}$ without constraints (9) and (10). By doing so, we are actually setting the $\boldsymbol{\pi}$ variables in (21) to 0. The strategy to be followed consists in decreasing the values of some $\boldsymbol{\nu}$ variables and at the same time increasing some $\boldsymbol{\pi}$ variables. Observe that for a given pair (i,t), if Δ units ($\Delta > 0$) are subtracted from ν_{it} then the first term of (24) decreases $\Delta\, Q_i$ units, whereas the second term increases $\Delta \sum_{j \in J: \lambda_{jt} - d_{jt}\nu_{it} - c_{ijt} \geq 0} d_{jt}$ units. Hence, it only compensates to decrease ν_{it} if $Q_i > \sum_{j \in J: \lambda_{jt} - d_{jt}\nu_{it} - c_{ijt} \geq 0} d_{jt}$.

Consider

$$k_j = \frac{\lambda_{jt} - c_{ijt}}{d_{jt}}, \quad j \in J \tag{25}$$

and let $k_{j_1} > k_{j_2} > \ldots > k_{j_p} > k_{j_{p+1}} > \ldots > k_{j_m}$ denote a non-increasing sequence where k_{j_p} is the smallest positive k_j value (if any exists) and $m \leq | J |$. The following result holds for the optimal solution of the dual subproblem (13)-(16) (without the $\boldsymbol{\pi}$ variables).

Proposition 3.1 *If the optimal solution of the dual of subproblem $SP_{\mathbf{z}}$ without constraints (9) and (10) has $\nu_{it} > 0$ for a given pair (i,t), $i \in I$ and $t \in T$, then $\nu_{it} = k_{j_1}$.*

Proof: Since $\boldsymbol{\pi} = 0$ and the feasibility of ν_{it} is imposed by conditions (14), it follows that the inequality $\lambda_{jt} - d_{jt}\,\nu_{it} \leq c_{ijt}$ must hold for every $j \in J$. Hence, $\nu_{it} \geq k_j$, $j \in J$. The equality holds due to the fact that the coefficients of ν_{it} in the dual objective function (13) are non-positive. Therefore, if some ν_{it} were greater than k_{j_1} we could set it equal to k_{j_1} without losing feasibility and increasing the dual objective function.

To solve (24) for a given pair (i,t), let us start by assigning k_{j_1} to ν_{it}. If $k_{j_1} \leq 0$ then actually $\nu_{it} = 0$ and no decrease should be allowed in this variable. Otherwise, we iteratively try to assign the values $k_{j_2}, \ldots, k_{j_p}, 0$ to ν_{it}. The procedure ends when the decrease obtained in the first part of (24) is smaller than the increase in the second part. For $1 \leq r \leq p$ define

$$\delta_r = \begin{cases} k_{j_r} - k_{j_{r+1}} & \text{if } r < p \\ \nu_{it} & \text{if } r = p \end{cases}$$

The following result holds for CP_{it} with $i \in I$ and $t \in T$.

Proposition 3.2 *Suppose that $\nu_{it} = k_{j_r}$ for some $1 \leq r \leq p$. If $Q_i > \sum_{\ell=1}^{r} d_{j_\ell t}$ and ν_{it} is decreased by δ_r then the objective function (24) decreases by $\delta_r \left(Q_i - \sum_{\ell=1}^{r} d_{j_\ell t}\right)$.*

Proof: Since $\nu_{it} = k_{j_r}$ for some r $(1 \le r \le p)$ it follows that $\nu_{it} \le k_{j_\ell}$ which by (25) leads to $\lambda_{j_\ell t} - d_{j_\ell t}\,\nu_{it} - c_{ij_\ell t} \ge 0$ for every $\ell = 1, \ldots, r$. Following a similar reasoning we conclude that $\lambda_{j_\ell t} - d_{j_\ell t}\,\nu_{it} - c_{ij_\ell t} < 0$ for every $\ell = r+1, \ldots, m$. If ν_{it} is decreased by δ_r then the term $\max\{0, \lambda_{j_\ell t} - d_{j_\ell t}\,\nu_{it} - c_{ij_\ell t}\}$ will actually increase by $\delta_r\, d_{j_\ell t}$, $\ell = 1, \ldots, r$. Consequently, the second term in (24) will grow by $\delta_r \sum_{\ell=1}^{r} d_{j_\ell t}$. In the case that $r < p$ this decrease in ν_{it} will bring $\lambda_{j_{r+1} t} - d_{j_{r+1} t}\,\nu_{it} - c_{ij_{r+1} t}$ up to zero but the terms $\lambda_{j_\ell t} - d_{j_\ell t}\,\nu_{it} - c_{ij_\ell t}$ will remain negative for $\ell = r+2, \ldots, m$. If $r = p$, a decrease of ν_{it} by δ_r will keep all the terms $\lambda_{j_\ell t} - d_{j_\ell t}\,\nu_{it} - c_{ij_\ell t}$ negative for $\ell = r+1, \ldots, m$. In both cases no further increase will occur in the second term of (24). Therefore, the global change in (24) induced by decreasing ν_{it} by δ_r is $-\delta_r\, Q_i + \delta_r \sum_{\ell=1}^{r} d_{j_\ell t}$. If $Q_i > \sum_{\ell=1}^{r} d_{j_\ell t}$ this change is negative, thus leading to a decrease in the objective function (24).

Algorithm 3.1 summarizes the procedure for modifying a variable ν_{it} (and π_{ijt}, $j \in J$) for a given pair (i, t), $i \in I$, $t \in T$. It makes use of the criteria established by Proposition 3.2. Note that the procedure is applied to the values of ν_{it} initially obtained by solving the dual of problem $SP_{\mathbf{z}}$ without considering constraints (9) and (10).

Algorithm 3.1 Procedure to modify ν_{it} and π_{ijt}, $j \in J$

Set $\pi_{ijt} = 0$ for each $j \in J$
Calculate $k_{j_1}, k_{j_2}, \ldots, k_{j_p}, \ldots, k_{jm}$
$r = 1$
while $\nu_{it} > 0$ **and** $r \le p$ **do**
  if $Q_i > \sum_{\ell=1}^{r} d_{j_\ell t}$ **then**
    $\nu_{it} = \nu_{it}$ - δ_r
    for $\ell = 1, \ldots, r$ **do**
      $\pi_{ij_\ell t} = \pi_{ij_\ell t} + \delta_r\, d_{j_\ell t}$
    $r = r + 1$
  else
    Stop

Algorithm 3.2 describes the complete strategy for enhancing an 'usual' cut, that is, a Benders cut.

In each iteration of the Benders algorithm two dual subproblems have to be solved: the first is problem (13)-(16) whose solution is considered for building the 'usual' cut; the second is the same problem but without the dual variables $\boldsymbol{\pi}$ (as a result of removing constraints (9) and (10) from the formulation of $DCFLP$). The optimal solution of the

Algorithm 3.2 Strategy for enhancing the 'usual' cut

Solve the dual problem of $SP_{\mathbf{z}}$ without constraints (9) and (10)
for $i \in I^c$ **do**
  while τ exists such that $z_{i\tau} = 1$ **do**
    for $t = \tau + 1, \ldots, n$ **do**
      if $\nu_{it} > 0$ **then**
        Modify ν_{it}, π_{ijt}, $j \in J$ (Algorithm 3.1)
for $i \in I^o$ **do**
  if $\sum_{\tau=2}^{n} z_{i\tau} = 0$ **then**
    for $t = 2, \ldots, n$ **do**
      if $\nu_{it} > 0$ **then**
        Modify ν_{it}, π_{ijt}, $j \in J$ (Algorithm 3.1)
  else
    while τ exists such that $z_{i\tau} = 1$ **do**
      for $t = 2, \ldots, \tau - 1$ **do**
        if $\nu_{it} > 0$ **then**
          Modify ν_{it}, π_{ijt}, $j \in J$ (Algorithm 3.1)

latter problem is needed to initialize the values of variables $\boldsymbol{\nu}$ in order to use Algorithm 3.1 to enhance the cut. At first glance it may seem that solving two dual subproblems will add to the overall computational effort. This situation can be handled by considering a slightly different relaxed master problem compared to the one suggested in Section 3.1. By simply removing constraints (9) and (10) from $DCFLP$, the resulting master problem is actually the same as (18)-(20) but without variables $\boldsymbol{\pi}$. Note that this can be done because the removed constraints are redundant. Hence, in this case the Benders algorithm remains the same as well as Algorithms 3.1 and 3.2. The only difference is that the first statement in Algorithm 3.2 is no longer necessary. As will be shown in the following section, this variant of the strategy for strengthening the 'usual' cut performed in general better in terms of overall computation time.

4 Computational results

In this section we report the results obtained with the proposed Benders decomposition method on a set of randomly generated test instances. In the next section the data generation process is described while Section 4.2 is dedicated to the presentation and discussion of the computational results.

4.1 Data generation

In total, 54 instances were randomly generated with varying sizes according to the number of time periods, the number of facilities, and the number of customers. Table 1 indicates the choices that were considered for these parameters. Using these combinations, we obtained instances with a number of binary variables between *40* and *700*, a number of continuous variables between *1000* and *75000*, and a number of constraints ranging from *165* to *2315*. Thus, small and medium-sized test instances were created.

$\mid T \mid$	$\mid I \mid$	$\mid I^c \mid$	$\mid J \mid$
5, 10, 15	10	3, 5, 8	20, 50, 100
5, 10, 15	20	8, 10, 16	50, 100
5, 10, 15	50	20, 25, 45	100

Table 1: Size of basic test instances.

Regarding the generation of all other parameters, Table 2 summarizes the relevant information.

Parameter	Value
$Q_i,\ i \in I$	$U[100, 300]$
$f_{it},\ i \in I^c,\ t \in T$	$Q_i \times U[8, 12.5]\,(1 + U^{t-1}[0, 10]\%)$
$f_{it},\ i \in I^o,\ t \in T \setminus \{1\}$	$Q_i \times U[8, 12.5]\,(1 + U^{t-1}[0, 10]\%)$
$g_{it},\ i \in I^c,\ t \in T \setminus \{n\}$	$Q_i \times U[7, 25]\,(1 + U^{t-1}[0, 10]\%)$
$h_{it},\ i \in I^o,\ t \in T \setminus \{1\}$	$Q_i \times U[9, 27]\,(1 + U^{t-1}[0, 10]\%)$
$d_{jt},\ j \in J,\ t \in T$	$[1, \frac{1}{\lvert J \rvert} \sum_{i \in I} Q_i]\,(1 + U^{t-1}[-10, 10]\%)$
$c_{ijt},\ i \in I^c,\ j \in J,\ t \in T$	$l_2(i, j) \times d_{jt}\,(1 + U^{t-1}[-10, 10]\%)$
$c_{ijt},\ i \in I^o,\ j \in J,\ t \in T \setminus \{1\}$	$l_2(i, j) \times d_{jt}\,(1 + U^{t-1}[-10, 10]\%)$

Table 2: Parameters selected for the random generation of the test instances.

We denote by $U[\alpha, \beta]$ the random generation of numbers in the interval $[\alpha, \beta]$ according to a Uniform distribution. Entries of the type $U[\alpha, \beta](1 + U^{t-1}[\gamma, \delta]\%)$ indicate that values between α and β were drawn from a Uniform distribution in period $t = 1$ ($t = 2$ if $i \in I^o$). In each subsequent period $t \in T \setminus \{1\}$ ($t \in T \setminus \{2\}$ if $i \in I^o$) the generated value was obtained from the current value applying a variation randomly generated according to a Uniform distribution in $[\gamma, \delta]$. The fixed facility costs F_{it} were determined as follows:

$$F_{it} = \begin{cases} \sum_{\tau=1}^{t} f_{i\tau} + g_{it} & \text{if } i \in I^c,\, t \in T \\ \sum_{\tau=t}^{n} f_{i\tau} + h_{it} & \text{if } i \in I^o,\, t \in T \setminus \{1\} \end{cases}$$

where $f_{i\tau}$ denotes the fixed cost of operating facility $i \in I$ in period τ, g_{it} indicates the fixed cost of closing the initially existing facility $i \in I^c$ at the end of period t (note that $g_{in} = 0$), and h_{it} represents the fixed cost of opening a new facility in site $i \in I^o$ at the beginning of period t. Table 2 describes the parameters of the Uniform distributions selected for the various types of fixed costs considered. Observe that the calculation of these costs depends on the capacity of each facility. Regarding the generation of the facilities and the customer locations, a 10×20 rectangle was considered and divided into $\mid I \mid$ longitudinal stripes, each having dimension $\frac{10}{|I|} \times 20$. In each stripe, one site corresponding either to an existing facility or to a new site was randomly positioned. The customer locations were randomly positioned in the overall rectangle. Using the coordinates of each facility-customer pair (i, j), the corresponding Euclidean distance, denoted by $l_2(i, j)$, was calculated and multiplied by the customer demand to obtain the cost of serving customer j from facility i in the first period, $j \in J$, $i \in I$. Hence, the distribution costs c_{ijt} satisfy the triangle inequality. Concerning the generation of these costs in the following periods, fluctuations between -10% and 10% were considered as shown in Table 2. Further details about the problem characteristics are described in Saldanha da Gama [26].

4.2 Summary of results

The proposed Benders algorithm was implemented in C^{++}. In each iteration, the relaxed master problem was solved with the standard mathematical programming software ILOG CPLEX 9.0 [8] embedded in ILOG CONCERT [7], a C^{++} interface. For comparison purposes, the branch-and-bound procedure of ILOG CPLEX 9.0 was also used to solve each problem instance. The formulation (1)-(8) of the $DCFLP$ with the additional constraints (11) was used. Tests with the formulations defined by (1)-(8), (1)-(11), and (1)-(10) were also performed but required larger computational effort and therefore, will not be discussed here (see Saldanha da Gama and Silva [27] for further details). CPLEX was stopped either when an integer solution within 0.1% of optimality was identified or when a time limit of twelve hours was attained. The 0.1% tolerance is acceptable in real-life situations due to the fact that there is always an error associated with data collection. The same tolerance was set in the Benders algorithm. Cordeau *et al.* [6] refer an error of at least 1% when estimates for costs, demands and capacities are collected. Finally, all tests were performed on a Pentium IV PC with a 3.2 GHz processor and 1 GB of RAM.

Tables 3 and 4 display the results obtained. The first four columns indicate the choices made for the parameters $| I |$, $| J |$, $| T |$ and $| I^c |$. Each table is divided into three parts, each corresponding to a set of test instances having the same total number of facilities and customers. In these tests the length of the planning horizon and the number of initially existing facilities were varied. The fifth column indicates the CPU time (in seconds) required by CPLEX to solve the corresponding test instance to optimality. The remaining four columns refer to two alternative variants of the Benders algorithm. The first variant - Strategy 1 - corresponds to solving two subproblems in each iteration, whereas in the second variant - Strategy 2 - one single dual subproblem is solved in each iteration (recall Section 3.2). For each variant, the overall CPU time (in seconds) and the total number of valid constraints ('# cuts') added to the relaxed master problem are indicated.

From Tables 3 and 4 it is clear that the Benders algorithm with the second strategy outperforms the other two procedures with respect to the computation time. The difference becomes more significant when the number of time periods increases (see, for example, the instance with 50 facilities, 100 customers, 10 periods, and 25 existing facilities in Table 4). The results also show that the performance of the three procedures is more sensitive to the number of time periods than to the number of customers. This is not surprising since an increase in $| T |$ leads to an increase in the number of binary variables. The same does not occur when $| J |$ grows.

Regarding the number of cuts, Strategies 1 and 2 are comparable, which means that solving only one dual subproblem in each iteration of the Benders algorithm (Strategy 2) does not have a major impact on the number of cuts generated. At first glance, this may seem surprising because in the first strategy additional dual information is considered and therefore, one could expect *stronger* cuts. However, the results show that in some way this information does not always help the enhancement procedure by actually yielding improved cuts. In only five instances did Strategy 1 produce less cuts than Strategy 2. Nevertheless, in three out of these five cases, Strategy 2 was faster.

$\mid I \mid$	$\mid J \mid$	$\mid T \mid$	$\mid I^c \mid$	CPLEX CPU (s)	Strategy 1 CPU (s)	Strategy 1 # cuts	Strategy 2 CPU (s)	Strategy 2 # cuts
			3	<1	<1	1	<1	1
		5	5	<1	<1	3	<1	3
			8	<1	<1	3	<1	3
			3	<1	<1	4	<1	3
10	20	10	5	<1	<1	4	<1	3
			8	<1	<1	3	<1	2
			3	7	25	6	1	3
		15	5	544	832	7	2	3
			8	<1	<1	3	<1	2
			3	<1	<1	2	<1	1
		5	5	<1	<1	3	<1	1
			8	<1	<1	5	<1	3
			3	6	1	3	<1	2
10	50	10	5	<1	<1	3	2	2
			8	6	7	9	<1	3
			3	85	27	8	7	3
		15	5	90	12	4	3	3
			8	296	97	9	19	3
			3	<1	<1	3	<1	2
		5	5	<1	<1	1	<1	1
			8	<1	<1	2	<1	1
			3	14	2	4	<1	3
10	100	10	5	5	2	3	<1	2
			8	1	1	2	<1	1
			3	104	4	4	2	3
		15	5	319	25	5	2	3
			8	116	5	5	1	3

Table 3: CPU time (seconds) for test instances with 10 facilities

Finally, it should be noted that a significant part of the CPU time required to run a test instance with Benders decomposition is spent on solving in each iteration a relaxed master problem, which is an integer linear program that we solved with CPLEX. However, even without tackling the master problems with a specially tailored method, the developed Benders algorithm clearly outperforms standard mathematical programming software. Note that CPLEX could not solve five of the largest test instances (see Table 4) within the prespecified time limit of twelve hours (43200 s). Nevertheless, the gaps reported by CPLEX upon termination were already very small, varying between 0.1% and 1%.

$\mid I \mid$	$\mid J \mid$	$\mid T \mid$	$\mid I^c \mid$	CPLEX CPU (s)	Strategy 1 CPU (s)	Strategy 1 # cuts	Strategy 2 CPU (s)	Strategy 2 # cuts
			8	<1	<1	2	<1	1
		5	10	<1	<1	2	<1	1
			16	1	2	1	<1	3
			8	46	19	8	1	3
20	50	10	10	154	160	10	<1	2
			16	47	73	8	7	6
			8	1202	191	12	245	3
		15	10	848	2116	19	96	8
			16	2808	1554	11	64	4
			8	<1	2	5	<1	4
		5	10	<1	1	3	<1	2
			16	2	6	11	<1	3
			8	74	12	6	2	3
20	100	10	10	34	17	9	2	3
			16	151	31	9	5	5
			8	383	118	7	10	4
		15	10	>43200	9418	9	2487	7
			16	4647	433	7	10	2
			20	<1	3	2	<1	2
		5	25	2	6	4	1	4
			45	31	37	16	4	11
			20	16516	113	6	326	28
50	100	10	25	>43200	1926	14	242	16
			45	272	127	14	17	10
			20	>43200	288	4	738	4
		15	25	>43200	109	3	888	4
			45	>43200	303	10	155	18

Table 4: CPU time (seconds) for test instances with 20 and 50 facilities

5 Conclusions

In this paper, a dynamic capacitated facility location problem was considered which includes not only phase-in location decisions (for setting up new facilities), as in classical facility location problems, but also phase-out location decisions (for closing existing facilities) throughout a given time horizon. By using a mixed integer linear programming model strengthened by a set of valid inequalities, and by exploiting the natural separation of the binary facility configuration variables and the continuous distribution variables, a primal Benders decomposition approach was developed to obtain optimal solutions within acceptable computational effort. To the best of our knowledge this is the first contribution towards applying such technique to solve dynamic phase-in/phase-out facility location problems. To accelerate the convergence of the method, a heuristic procedure was proposed to enhance the quality of the usual Benders cuts. This method makes use of the existence of multiple optimal solutions of the dual subproblem, which means that a number of valid alternative Benders cuts is available.

The computational experiments suggest that the proposed Benders decomposition approach is an efficient method that can be used for solving large facility location problems. The largest problems with 15 planning periods, 50 facilities and 100 customers were solved within 5 minutes of CPU time whereas an off-the shelf solver like CPLEX could not solve such problems within 12 hours. This significant reduction in computation times was achieved even though the relaxed Benders master problems, which are integer linear problems, were solved with CPLEX. Hence, further research will be directed towards the development of a new method that explores the special structure of these problems. Meta-heuristics such as variable neighbourhood search seem to be a promising approach for tackling the master problems. This step will increase the possibility to solve realistically sized problems in a reasonable amount of time, and thus provide the decision-maker with a tool to redesign the logistics networks and to re-evaluate alternative network configurations on a regular basis. Another interesting direction for further research would be the extension of the model and the solution method to handle cases with multiple products and multiple echelons of facilities. Recently, Cordeau *et al.* [6] applied Benders decomposition to a comprehensive, yet static, network design problem considering these aspects. Their promising results encourage the generalization of our approach to this more complex class of problems.

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