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The dynamics of financial stability in complex networks

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Abstract. We address the problem of banking system resilience by applying off-equilibrium statistical physics to a system of particles, representing the economic agents, modelled according to the theoretical foundation of the current banking regulation, the so called Merton-Vasicek model. Economic agents are attracted to each other to exchange 'economic energy', forming a network of trades. When the capital level of one economic agent drops below a minimum, the economic agent becomes insolvent. The insolvency of one single economic agent affects the economic energy of all its neighbours which thus become susceptible to insolvency, being able to trigger a chain of insolvencies (avalanche). We show that the distribution of avalanche sizes follows a power-law whose exponent depends on the minimum capital level. Furthermore, we present evidence that under an increase in the minimum capital level, large crashes will be avoided only if one assumes that agents will accept a drop in business levels, while keeping their trading attitudes and policies unchanged. The alternative assumption, that agents will try to restore their business levels, may lead to the unexpected consequence that large crises occur with higher probability.

1 Introduction

The well-being of humankind depends crucially on the financial stability of the underlying economy. The concept of financial stability is associated with the set of conditions under which the process of financial intermediation (using savings from some economic agents to lend to other economic agents) is smooth, thereby promoting the flow of money from where it is available to where it is needed. This flow of money is made through economic agents, commonly called 'banks', that provide the service of intermediation and an upstream flow of interest to pay for the savings allocation. Because the flow of money that ensures financial stability occurs on top of a complex interconnected set of economic agents (network), it must depend not only in individual features or conditions imposed to the economic agents but also on the overall structure of the entire economic environment. The role of banking regulators is to protect the flow of money through the system by implementing rules that insulate it against individual or localised breaches that happen when a bank fails to pay back to depositors. However, these rules do not always take into account the importance of the topological structure of the network for the global financial stability. In this paper, we will present quantitative evidence that neglecting the topological network structure when implementing financial regulation may have a strong negative impact on financial stability.

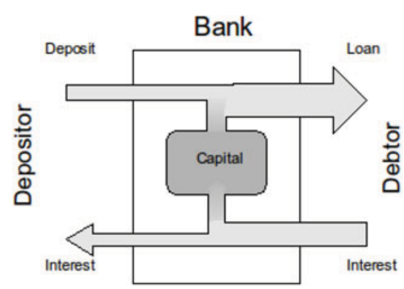

Fig. 1. Illustration of a bank 'apparatus' for money flow. A bank lends money to debtors using money from depositors and also its own money, the capital. In return debtors pay interest to the bank, which keeps a part to itself and pays the depositors back.

The event of not paying back the money owed is called 'default'. In order that downstream defaults do not generate the default of a particular bank, each bank holds an amount of money as a reserve for paying back its depositors. In other words, a part of the money one bank sends downstream is its own money. This share of own money is called 'capital' (see Fig. 1). Looking to one single bank, if it has a large amount of capital, one reasonably expects that the bank will also cover a proportionally large debtor default, guaranteeing the deposits made by its depositors. On the contrary, if the capital level of the bank is small, a small debtor default is sufficient to put the bank with no conditions for guaranteeing the money of all its depositors.

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Loosing such conditions, the bank enters a situation called bankruptcy or insolvency. Usually, bank regulators base their rules in such arguments.

In 1988, a group of central bank governors called the Basel Committee on Banking Regulation unified the capital level rules that were applied in each of the member countries and defined a global rule to protect the banking system that was becoming global at the time [1]. Roughly speaking, these rules imposed a minimum capital level of 8% without any empirical reason. A few years later the accord came under criticism from market agents who felt that it did not differentiate enough between the various debtors, i.e. between the entities whom the bank lends money, and a second version of the accord [2] was finished in 2004 to become effective in 2008. In this second version, banks were allowed to use the Merton-Vasicek model [3] based on the Value-at-Risk paradigm(VaR) [4] to weight the amount of lent money in the calculation of the necessary capital according to the measured risk of the debtor. Thus the 8% percentage was now calculated over the weighted amount and not over the total amount. This version of the accord become effective at the beginning of the 2008 financial turmoil; with regulators under severe criticism from governments and the media, in 2010 the Committee issued a new version [5] tightening capital rules.

At the same time, since the beginning of the 2000s the academic community has been very critical of the capital rules, particularly because the VaR paradigm, on which such rules are based, assumes that returns are normally distributed and "does not measure the distribution or extent of risk in the tail, but only provides an estimate of a particular point in the distribution" [6]. In fact, there is a huge amount of evidence [7–10] that the returns of economic processes are not normally distributed, having typically heavy tails. According to the Central Limit Theorem [11], if returns are heavy-tail distributed, then the underlying random variables have infinite variance or a variance of the order of the system size [12]. In economic systems, random variables are related to measurements taken from economic agents. Thus, the infinite variance results from long-range correlations between the economic agents. We will argue that this single fact compromises the stability of the flow and brings into question the effectiveness of capital level rules.

Physics, and in particular statistical physics, has long inspired the construction of models for explaining the evolution of economies and societies and for tackling major economic decisions in different contexts [13,14]. The study of critical phenomena and multi-scale systems in physics led to the development of tools that proved to be useful in non-physical contexts, particularly in financial systems. One reason for this is that fast macroeconomic indicators, such as principal indices in financial markets, exhibit dynamical scaling, which is typical of critical physical systems [15].

In this paper we will address the problem of the financial stability using statistical physics models that explain the occurrence of large crises, in order to show that the resilience of the banking system is not necessarily improved by raising capital levels. Our findings have a concrete social importance, since capital is the most expensive money a bank can provide to its debtors. Capital belongs to the shareholders, who bear the risk of the business and keep the job positions. So it must be remunerated above the money from depositors who do not bear these risks. Consequently, more capital means more costs on the flow of the money and, in the end, more constraints to economic development.

We start in Section 2 by describing an agent-based model [14] which enables us to generate the critical behaviour observed in economic systems. In Section 3 we describe the observables that account for the economic properties of the system, namely the so-called overall product and business level [16]. Furthermore, the agent-based model as well as the macroscopic observables, are discussed for the specific situation of a network of banks and their deposits and loans. One important property in financial banking systems is introduced, namely the minimum capital level, defined here through the basic properties of agents and their connections. In Section 4 we focus on the financial stability of the banking system, showing that raising capital levels promotes concentration of economic agents if the economic production remains constant and it destroys economic production if that concentration does not occur. Finally, we present specific situations where each agent seeks the stability of its economic production after a raise in capital levels, leading to a state of worse financial stability, i.e. a state in which large crises are more likely to occur. In Section 5 we draw the conclusions.

2 Minimal model for avalanches of financial defaults

The model introduced in this section is based on a fundamental feature that human beings have developed in their individual behaviour, through natural selection, in order to be able to fight environmental threats collectively. It is called *specialisation* [17], and describes the tendency individuals have, when living in communities, to concentrate on one, or at most a few, specific tasks. Each individual does not need to do everything to survive, just to concentrate on a few tasks that he/she can do better for all the other individuals. Everything else he or she will get from other specialised elements of the community. Thus, specialisation leads to optimisation, enabling the entire community to accomplish goals otherwise unattainable. However, it also implies that individuals now need to exchange what they do, so that all have everything they need for survival. The set of all task and product exchanges between individuals is what we usually call *Economy.* Consequently, when building an economic system, a reasonable approach is to take agents which are *impelled* to exchange some product through a network of trades between pairs of agents. In this scope, let us assume that the economic environment is composed of elementary particles called 'agents' and all phenomena occurring in it result from the interaction of those particles. Let us also assume that agents are attracted to interact, exchanging an observed quantity that takes the form of money, labour or other effective means used in the exchange. This type of model where the decision concerning an exchange is made by the exchanging agents alone is called a "free-market economy".

We represent these interactions or trades between agents through economic connections, and call the exchanged quantity 'economic energy'. Though the "energy" used here is not the same as physical energy, we will use the term in the economical context only. Notice, however, that human labour is assumed to be "energy" delivered by one individual to those with whom he/she interacts, which reward the individual with an energy that he/she accumulates. The balance between the labour ("energy") produced for the neighbours and the reward received from them may be positive (agent profits) or negative (agent accumulates debt). For details see reference [18]. This analogy underlies the model introduced in the following, where we omit the quotation marks and consider entities more general than individual, which we call agents. Agent-based models of financial markets have been intensively studied, see e.g. references [14,19] and references therein.

Economic connections between two agents are in general not symmetric and there is one simple economic reason for that: if a connection were completely symmetric there would be no reason for each of the two agents to establish an exchange. In several branches of Economics we have different examples of these asymmetric economic connections like production/consumption, credit/deposit, a labour relation, repo's, swaps, etc. In the next section, we will focus on a specific connection, namely in credit/deposit connections.

Since each connection is asymmetric we distinguish the two agents involved by assigning two different types of economic energy. Hence, let us consider two connected agents, i and j, where i delivers to j an amount of energy W_{ij} and receives an amount $E_{ij} \neq W_{ij}$ in return. We call these connections the outgoing connections of agent i. The connections where agent i receives from j an amount of energy W_{ji} and delivers in return E_{ji} we call incoming connections.

The energy balance for agent i in one single trade connection is, from a labour production point of view, $U_{ij} = W_{ij} - E_{ij}$. Having two different types of energy, we choose W_{ij} as the reference to which the other type $E_{ij} = \alpha_{ij} W_{ij}$ is related through the coefficient α_{ij} (see Eq. (2) below). Without loss of generality, we consider that one connection corresponds to the delivery of one unit of energy, $W_{ij} = 1$, yielding:

$$U_{ij} = 1 - \alpha_{ij}. \tag{1}$$

In order to implement the model, we need to define the form of the coefficient α_{ij} in equation (1), which is a connection property. The amount of energy E_{ij} that one agent i gets in return for a delivered amount W_{ij} can be taken as a price which depends on the rules of supply and demand. An agent delivering energy to many neighbours tends to impose a higher price on them. Similarly, an agent receiving energy from many other neighbours will induce a reduction in the price imposed by its creditors. These principles can be incorporated in a simple *Ansatz* as:

$$\alpha_{ij} = \frac{2}{1 + \mathrm{e}^{-(k_{out,i} - k_{in,j})}} \tag{2}$$

where $k_{out,i}$ is the number of outgoing connections of agent i and $k_{in,j}$ is the number of incoming connections of neighbour j. For $\alpha_{ij} > 1$ the energy provided by agent i to agent j is 'paid' by j above the amount of energy agent i delivers. Thus, agent i profits from this connection and gains a certain amount of energy, $U_i > 0$. For $\alpha_{ij} < 1$ the opposite occurs. From equation (2) one easily sees that α_{ij} is a step-function with average value one and very small derivatives in the asymptotic limits $k_{in,j} \gg k_{out,i}$ and $k_{in,j} \ll k_{out,i}$. Furthermore, in this latter limit $k_{in,j} \ll k_{out,i}$, the value of α_{ij} could in principle be any finite value larger than one. However, to guarantee symmetry between the situation when agent i profits from agent j, and that when agent j profits from agent i, we consider the range $\alpha_{ij} \in [0, 2]$, yielding $\alpha_{ij} = 2$ for $k_{in,j} \ll k_{out,i}$.

Such energy transactions have an Economics analogue according to basic principles [17]: a large (small) $k_{in,j}$ indicates a large (small) supply for agent j and a large (small) $k_{out,i}$ indicates a large (small) demand of agent i. Thus, the difference $k_{out,i} - k_{in,j}$ measures the balance between the demand of an agent i and the supply of its neighbour j. α_{ij} saturates for large positive and negative differences in order to guarantee the price to be finite.

The definition of α_{ij} in equation (2) is not uniquely determined by these economic requisites. Similar functions such as the arc-tangent or the hyperbolic tangent have been used in this context [20]. The main findings of this manuscript are not sensitive to the choice of functional dependency of α_{ij} as long as it is a step-function of $k_{in,j} - k_{out,i}$.

In the model described above, we disregard the economic details of agents and connections, keeping the model as general as possible. Still, this generalisation is not different in its essence from the one accountants must use to provide a common report for all sorts of business, with the difference that they use monetary units and we use dimensionless energy units.

Because each agent typically has more than one neighbour, the total energy balance for agent i is given by

$$U_i = \sum_{\text{all neighbours}} U_{ij} = \sum_{j \in \nu_{out,i}} (1 - \alpha_{ij}) + \sum_{j \in \nu_{in,i}} (\alpha_{ji} - 1) \tag{3}$$

where j runs over all neighbours of agent i, and $\nu_{out,i}$ and $\nu_{in,i}$ are, respectively the outgoing and incoming vicinities of the agent.

This total energy balance U_i is related to the well-known financial principle of net present value (NPV) [21]: When an agent holds a deposit he or she supposedly pays for it (by definition) and most (but not all) accounting standards [22] assume it as a negative entry on the accounting balance. Here, we model deposits as a set of incoming connections from the same agent in which all associated cash-flows were already discounted. In this way, if we could think of a balance sheet totally built with NPV's we would be near U_i.

As we noted previously, economic energy is related to physical energy in the sense that the agents must absorb finite amounts of physical energy from the environment to deliver economic energy. Consequently, the economic energy balance U_i of agent i must be finite. The finiteness of U_i for each agent is controlled by a threshold value, below which the agent is no longer able to consume energy from its neighbours, i.e. below which it loses all its incoming connections. Furthermore, since this threshold reflects the incoming connections, it should depend on how many incoming connections our agent has. With such assumptions, we introduce the quantity

$$c_i \equiv \frac{U_i}{\sum_{j\in\nu_{in,i}}(\alpha_{ji}-1)} \tag{4}$$

for ascertaining if the agent is below a given threshold c_{th} or not. We call this quantity c_i the 'leverage' of agent i. Unlike we did previously in reference [18], here U_i is divided by the total product of the incoming connections solely and not by the 'turnover'. This choice is made to be in line with the way banking regulators define leverage. Still, this alternate definition does not change the critical behaviour observed in our model and previously reported [18]. For the case that the mean-field approximation $\alpha_{ij} \sim \langle\alpha\rangle$ holds, the leverage c_i depends exclusively on the network topology, yielding $c_i = \frac{k_{out,i}}{k_{in,i}} - 1$ [18].

Leverage has a specific meaning in Economics, which related to the quantity c_i: it measures the ratio between own money and total assets [21]. Thus, each agent has a leverage c_i which varies in time and there is a threshold c_{th} below which the agent 'defaults' or goes bankrupt, losing its incoming connections with its neighbours. Since the bankrupted agent is connected to other agents, the energy balances must be updated for every affected agent j. Bankruptcy leads to the removal of all incoming connections of agent i, reducing the consumption of the bankrupted agent to a minimum, i.e. keeping one single consumption connection, $k_{in,i} = 1$. This situation implies that agent i and its neighbours j should be updated as follows:

$$c_i \to k_{out,i} - 1 \tag{5a}$$
$$k_{out,j} \to k_{out,j} - 1 \tag{5b}$$
$$c_j \to c_j - \frac{1}{k_{in,j}}. \tag{5c}$$

We keep the agent with one consumption connection in the system also to avoid the divergence of c_i as defined in the context of financial regulation [1,2,5]. Such a minimum consumption value has no other effect on the problem we will be dealing with in the next section.

The bankruptcy of i leads to an update of the energy balance for neighbour j, which may then also go bankrupt, and so on, thereby triggering a chain of bankruptcies henceforth called an 'avalanche'. See illustration in Figure 2.

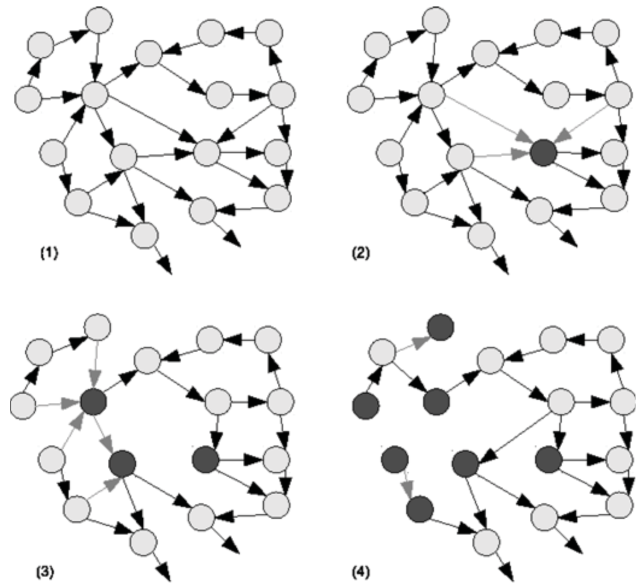

Fig. 2. Illustration of a bankruptcy avalanche. Each arrow points to the agent for which it is an incoming connection. Any agent in the economic environment is part of a complex network (1), and is susceptible to go bankrupt, which will destroy its incoming connections (2). Consequently, new energy balances must be updated for the affected neighbours, whose leverage can go over the threshold (3). Since these neighbours also have neighbours of their own, connection will continue to be destroyed until all agents again have a leverage above the threshold.

The concepts of leverage and leverage threshold are used by Merton [23] and Vasicek [3] in their credit risk models, which are the theoretical foundation for the Basel Accords [1,2,5]. Namely, Merton assumed this threshold for pricing corporate risky bonds using a limit on debt-equity ratio and Vasicek generalised it to a "debtor wealth threshold" below which the debtor would default on a loan.

3 Macroproperties: overall product and business level

Let us consider a system of L interconnected agents which form the environment where each agent establishes its trades. We call henceforth this environment the operating neighbourhood. We can measure the total economic energy of the system by summing up all outgoing connections to get the overall product U_T, namely

$$U_T = \sum_{i=1}^{N} \sum_{j\in\mathcal{V}_{out,i}} (1-\alpha_{ij}), \tag{6}$$

where $\mathcal{V}_{out,i}$ is the outgoing vicinity of agent i, with $k_{out,i}$ neighbours. The quantity U_T varies in time and its evolution reflects the development or failure of the underlying economy. Instead of U_T, we consider the relative variation $\frac{dU_T}{U_T}$, also known as 'return' in a financial context. We can also measure the average business level per agent, defined as the moving average in time of the overall product:

$$\Omega = \frac{1}{L}\frac{1}{T_S}\int_t^{t+T_S} U_T(x)dx \quad (7)$$

where T_S is a sufficiently large period for taking time averages. Similar quantities are used in Economics as indicators of individual average standard of living [16]. In the continuum limit, the time derivative of the business level Ω gives the overall product uniformly distributed over all agents.

At each time step a new connection is formed, according to the standard preferential attachment algorithm of Barabási-Albert [24]: for each connection created one agent is selected using a probability function based on its previous outgoing connections, expressed as

$$P(i) = \frac{k_{out,i}}{\sum_{l=1}^{L} k_{out,l}} \quad (8)$$

and one other agent is selected by an analogous probability function built with incoming connections. Such a preferential attachment scheme is associated with power-law features observed in the Economy long ago [25,26] and is here motivated by first principles in economics that agents are impelled to follow: an agent having a large number of outgoing connections is more likely to be selected again to have a new outgoing connection, and likewise for incoming connections.

As connections are being created, a complex network of economic agents emerges and individual leverages (see Eq. (4)) are changing until eventually one of the agents goes bankrupt ($c_i < c_{th}$) breaking its incoming connections and changing the leverage of its neighbours, who might also go bankrupt and break their incoming connections and so on. See Figure 2. This avalanche affects the total overall product, equation (6), because the dissipated energy released during the avalanche is subtracted. This total dissipated energy is given by the total number of broken connections, and measures the 'avalanche size', denoted below by s. Since the avalanche can involve an arbitrary number of agents, and is bounded only by the size of the system, the distribution of the returns $\frac{dU_T}{U_T}$ will be heavy-tailed, as expected for an economic system. See Figure 5 below.

Until now we have been dealing with generic economic agents that make generic economic connections between each other. No particular assumption has been made besides that they are attracted to each other to form connections by the mechanism of preferential attachment and that the economic network cannot have infinite energy. From this point onward, we will differentiate some of these agents, labeling them as 'banks'. To this end we fix the nature of their incoming and outgoing connections: the incoming connections are called 'deposits', the outgoing connections are called 'loans'. We should emphasize that we are not singling out this kind of agent from the others. Banks are modelled as economic agents like any other. We have only named its incoming and outgoing connections, which we could also do for all the remaining agents, as consuming/producing, salary/labour, pension/contribution, etc., to model every single business we could think of. We are choosing this particular kind of agent because banks are the object of banking regulation and the aim of financial stability laws.

The threshold leverage c_{th} for one bank represents its 'minimum capital level'. The capital of one bank is really an amount of incoming connections, which are equivalent to deposits, because shareholders are also economic agents. This means that the 'minimum capital level' in the model will be much higher than in real bank markets because we are disregarding shareholders and adding the remaining energy deficit to fulfill c_{th}. Therefore, we cannot map directly the levels obtained in the model onto the levels defined in banking regulation. We can, however, uncover the behaviour of economic agents in scenarios difficult to reproduce without such a model.

4 Raising the minimum capital level

In this section we use the model described above in different scenarios, i.e. for different sizes of the operating neighbourhood and different minimum capital levels. From equation (4) one sees that the leverage of one agent is always larger than -1. Since we deal with bankruptcy we are interested in negative values of c_{th}, which reduces the range of leverage values to $[-1,0]$. Our simulations showed that a representative range of values for both the threshold and the size L of the operating neighbourhood is $[-0.72, -0.67]$ and $[500, 2000]$ respectively. For each pair of values (L, c_{th}) the system evolves until a total of 1.5×10^6 connections are generated. We discard the first 10^5 timesteps which are taken as transient.

Figures 3 and 4 illustrate the evolution of the overall product U_T and business level Ω for a situation in which the minimum capital level is raised, while keeping the size of the operating neighbourhood constant. The solid line shows the initial situation with lower minimum capital level and the dashed line the final situation with higher minimum capital level. From Figure 3 we can see that if the size of the operating neighbourhood is kept constant, the quasi-stationary level of the overall product does not significantly change.

Following this observation we next investigate the evolution of the return distribution for U_T, considering an increase of the minimum capital level at constant size L of the operating neighbourhood. To this end we compute the cumulative size distribution of avalanches, i.e. the fraction $P_c(s)$ of avalanches of size larger than s. Numerically, the size s of an avalanche is found by summing all connections destroyed during that avalanche. The value of $P_c(s)$ is then obtained by identifying the avalanches whose size is greater than s.

Figure 5 shows the cumulative size distribution of avalanches for different minimum capital levels, keeping

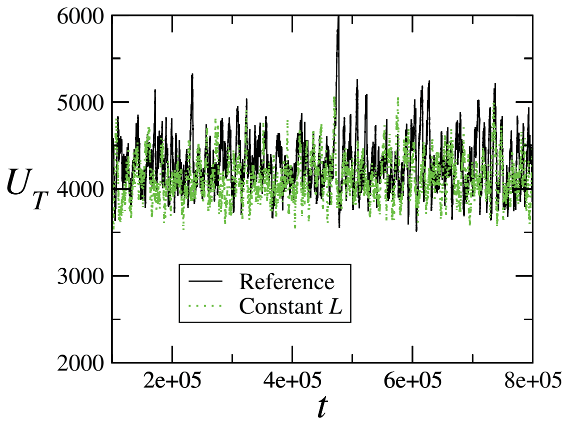

Fig. 3. (Color online) Illustration of the effect of raising the minimum capital level on the overall product U_T, at constant $L = 1500$. Raising c_{th} from -0.71 (solid line) to -0.69 (dotted line) does not significantly change the overall product.

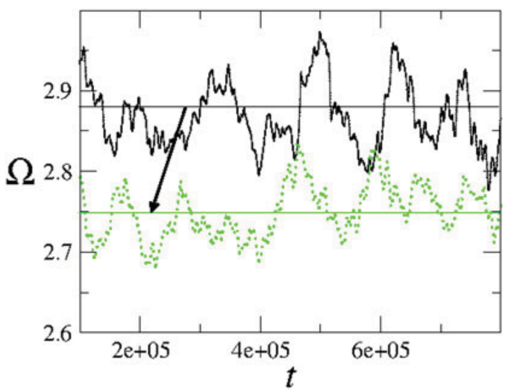

Fig. 4. (Color online) Illustration of the effect of raising the minimum capital on the business level at constant $L = 1500$. Raising c_{th} from -0.71 (solid line) to -0.69 (dotted line) decreases the business level from $\Omega = 2.88$ to $\Omega = 2.75$.

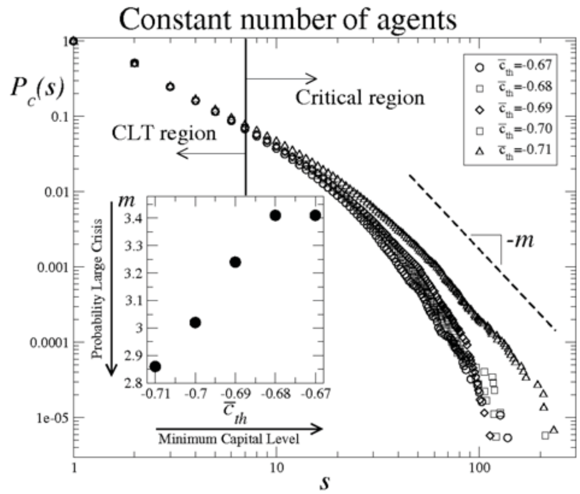

Fig. 5. Avalanche (crises) size distributions for different scenarios of minimum capital level, keeping the operating neighbourhood unchanged for each agent. The different distributions match at small sizes, in the region where the central limit theorem (CLT) holds, and deviate from each other for larger crises (critical region). In the critical region one observes (inset) that increasing the minimum capital level decreases the probability for a large avalanche to occur, which supports the intentions of the Basel III accords. However, in this scenario one assumes that each bank will have a simultaneous decrease of their business level (see text and Fig. 4). A more natural scenario would be one where each bank reacts to the rise in the minimum capital level in such a way as to keep its business level constant, which leads to a completely different crises situation (see Fig. 8).

$L = 2000$. For small avalanche sizes, the Central Limit Theorem holds [12] and thus all size distributions match independently of the minimum capital level. For large enough avalanches ('critical region'), the size distributions deviate from each other, exhibiting a power-law tail $P_c(s) \sim s^{-m}$ with an exponent m that depends on the minimum capital level c_{th} (inset). As expected [27], the exponent found for the avalanche size distribution takes values in the interval $2 < m < 7/2$.

As can be seen in the inset of Figure 5 the exponent increases in absolute value for larger minimum capital levels, indicating a smaller probability for large avalanches to occur. However, this scenario occurs only when the size of the operating neighbourhood is kept constant and, as shown in Figure 4, the increase of the minimum capital level is also accompanied by a decrease of the business level. This means that each agent has less economic energy or, in current language, is poorer.

Assuming that agents do not want to be poorer despite regulatory constraints, and therefore try to keep their business levels constant (Fig. 6), a natural reaction against raising the minimum capital level is to decrease the number of neighbours with whom the agent establishes trade connections, i.e. to decrease the size of the operating neighbourhood (Fig. 7). In Economics this is called a concentration process [28], which typically occurs when the regulation rules are tightened up. In such a scenario where the size of the operating neighbourhood is adapted so as to maintain the business level constant, the distributions plotted in Figure 5 are no longer observed. In particular, the exponent m does not increase monotonically with the minimum capital level as we show next.

Figure 8a shows the critical exponent m and the business level per agent Ω as functions of the minimum capital level c_{th} and the operating neighbourhood size L. For easy comparison, both quantities are normalized in the unit interval of accessible values.

The critical exponent shows a tendency to increase with both the minimum capital level and the operating neighbourhood size. The business level, on the other hand, decreases when the minimum capital level or the neighbourhood size increase. Considering a reference state F_0 with $c_{th,0}$, L_0 and Ω_0 there is one isoline of constant minimum capital level, $\Gamma^0_{c_{th}}$, and another of constant operating neighbourhood size, Γ^0_L, crossing at F_0. Assuming a

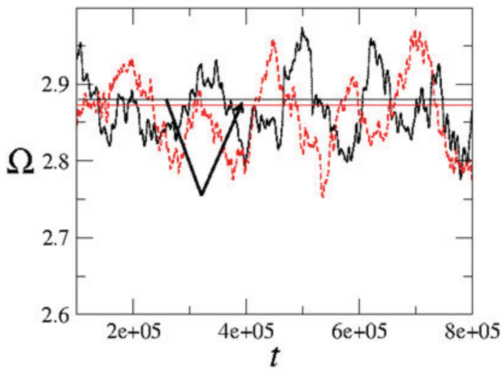

Fig. 6. (Color online) Unlike in Figures 3 and 4 it is possible to raise the minimum capital level c_{th} from -0.71 (solid line) to -0.69 (dashed line), while keeping the business level constant. In the case plotted, $\Omega \sim 2.88$.

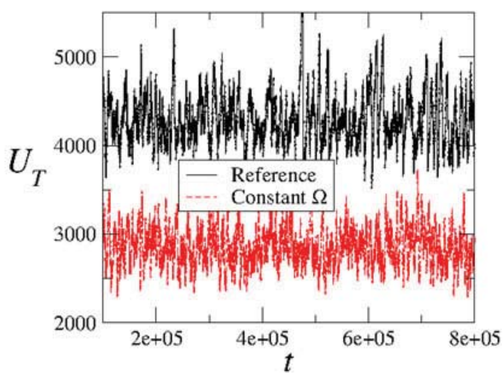

Fig. 7. (Color online) Keeping the business level constant at $\Omega \sim 2.88$ and raising the minimum capital level from -0.71 (solid line) to -0.69 (dashed line) leads to a decrease of the operating neighbourhood, which is reflected in a lower overall product.

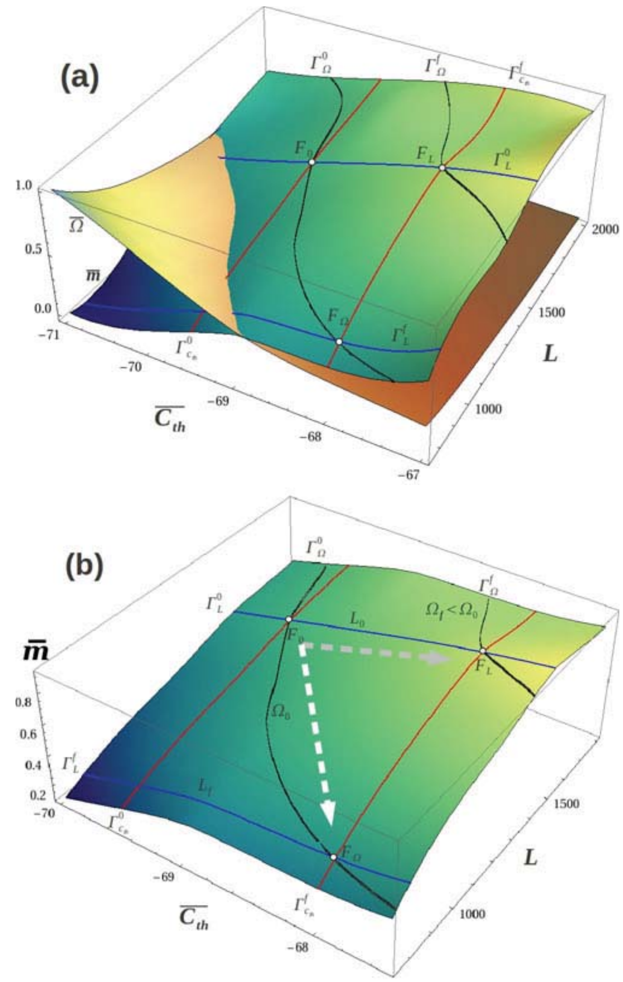

Fig. 8. (Color online) (a) Normalized critical exponent $\bar{m}$ and business level Ω as functions of the minimum capital level c_{th} and system size L. For an initial financial state F_0, an increase of the minimum capital level takes the system along one of the infinitely many paths between the initial and final isolines at constant minimum capital level, $\Gamma^0_{c_{th}}$ and $\Gamma^f_{c_{th}}$ respectively. (b) If such a path follows the isoline at constant system size, Γ^0_L, the critical exponent increases and thus the probability for large avalanches decreases. Simultaneously however, its business level decreases ($\Omega_f < \Omega_0$), which runs against the natural intentions of financial agents. On the contrary, if the path is along the isoline at constant business level, Γ^0_Ω, as one naturally expects the financial agents would do, the critical exponent does not change significantly, meaning that large financial crises may still occur with the same probability as before (see text).

transition of our system to a larger minimum capital level at isoline $\Gamma^f_{c_{th}}$ while keeping L constant, i.e. along the isoline Γ^0_L, one arrives at a new state F_L with a larger critical exponent, which means a lower probability for large avalanches to occur, as explained above. However in such a situation the new business level Ω_f is lower than the previous one Ω_0.

On the contrary, if we assume that the transition from F_0 to the higher minimum capital level occurs at constant business level, i.e. along the isoline Γ^0_Ω, one arrives to a state F_Ω on the isoline $\Gamma^f_{c_{th}}$ for which the critical exponent is not necessarily smaller than for the initial state.

From economical and financial reasoning, one typically assumes that, independently of external directives, under unfavourable circumstances economical and financial agents try, at least, to maintain their business level. This behaviour on the part of agents leads to a situation which contradicts the expectations of the Basel accords and raises the question of whether such regulation will indeed prevent larger avalanches from occurring again in the future. To illustrate this, Figure 8b shows a close-up of the m-surface plotted in Figure 8a.

For the reference state F_0 one finds an exponent $m = 2.97 \pm 0.18$. An increase of the minimum capital level at constant operating neighbourhood size (state F_L) yields $m = 3.34 \pm 0.09$, while increasing the minimum capital level at constant business level (state F_Ω), yields $m = 2.79 \pm 0.09$, which corresponds to a significantly higher probability that large avalanches will occur.

5 Discussion and conclusion

In summary, raising the minimum capital levels may not necessarily improve banking system resilience. Resilience may remain the same if banks go after the same business levels, as one should expect, according to economic reasoning. Indeed, since business levels are part of the achievement of any economic agent that enters a network of trades, each agent will try, at least, to maintain this level, independently of regulatory constraints.

Furthermore, our findings can solve the apparent contradiction between the credit risk models that serve as the theoretical foundation for bank stability accords and the definition of capital levels. In fact, bank stability accords impose on banks an adapted version of Merton-Vasicek model [3] in which it is assumed that each agent has a leverage threshold above which it defaults on credit. The assumption of this threshold combined with a first principle of Economics – that the Economy emerges from the exchanges between agents – naturally leads to an interplay between agents that can propagate the effect of one default throughout the entire economic system.

Economic systems have long-range correlations and heavy-tailed distributions that are not compatible with a linear assumption that raising individual capital levels will lead to stronger stability. Because of the interdependency, this assumption is probably valid only in two situations: when it is impossible for an individual to default; and when individuals behave independently from each other (random trade connections). Both situations do not occur in real economic systems.

These findings can inform the recent governmental measures for dealing with the effects of the 2008 financial crises. In particular, governments have shown [5] a tendency for imposing a higher capital investment from banks. If the threshold is increased, while the total amount of trade remains constant, there will be fewer trade connections between the banks and their clients, which leads to smaller avalanches in the evolution of the financial network. On the other hand, if the total amount of trade is assumed to grow, following the rise in minimum capital, the probability of greater avalanches will also increase to the level where it was before or even to a higher level.

The scale-free topology of the economic network plays a major role in the determination of the size distribution of the avalanches. At the same time, the scale-free topology emerges naturally from the rules introduced, which are motivated by economic reasoning, namely the principles of demand and supply. Still, one could argue that for bank regulation purposes, a different (imposed) topology for the connections between financial agents would help to prevent large crises. For example, if the economic network is structured as a random Erdös-Rényi network [29], in which every economic agent has the same probability of being chosen to form an economic connection, the system would not have avalanches. In such a model, since connections are equally distributed throughout the system, all agents would have statistically the same balance. In other words, for each bankruptcy the expected number of child bankruptcies in the avalanche would have either zero size or the size of the system. Thus, with Erdös-Rényi topology, one expects still the danger of triggering such a large chain of insolvencies able to collapse the entire system.

Directives more oriented to the connection topology emerging in the financial network could be a good alternative. Interestingly, although controversial, our claims point in the direction of IMF reports in November 2010 [30], where it is argued that rapid growth in emerging economic periods can be followed by financial crises, and also to recent theoretical studies on the risk of interbank markets [31,32]. Indeed the recent IMF Memorandum on Portuguese economic policy [33] already includes directives that reveal IMF's concern not only with tuning capital buffers and other local properties but also with monitoring the banking system as a whole, and in particular keeping track of the financial situation of the largest banks in the network. We believe that such global networking measures are much more trustworthy than local ones.

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