

SEMINÁRIO

ANÁLISE E EQUAÇÕES DIFERENCIAIS

28 de Novembro | 13h30 | sala 6.2.33

Some inequalities for fractional Laplacians

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Abstract:

Let Ω be a bounded domain with smooth boundary. We compare two natural types of fractional Laplacians $(-\Delta)^s$, namely, the “Navier” and the “Dirichlet” ones. We denote their quadratic forms by $Q_{s,\Omega}^N$ and $Q_{s,\Omega}^D$, respectively.

Theorem 1. Let $s > -1$, $s \notin \mathbb{N}_0$. Then for $u \in \text{Dom}(Q_{s,\Omega}^D)$, $u \neq 0$, the following relations hold:

$$Q_{s,\Omega}^N[u] > Q_{s,\Omega}^D[u], \quad \text{if } 2k < s < 2k+1, \quad k \in \mathbb{N}_0;$$

$$Q_{s,\Omega}^N[u] < Q_{s,\Omega}^D[u], \quad \text{if } 2k-1 < s < 2k, \quad k \in \mathbb{N}_0.$$

Theorem 2. Let $0 < |s| < 1$, and let $u \in \text{Dom}(Q_{s,\Omega}^D)$, $u \geq 0$, $u \neq 0$. Then the following relations hold (all inequalities are understood in the sense of distributions):

$$(-\Delta_\Omega)_N^s u > (-\Delta_\Omega)_D^s u, \quad \text{if } 0 < s < 1;$$

$$(-\Delta_\Omega)_N^s u < (-\Delta_\Omega)_D^s u, \quad \text{if } -1 < s < 0.$$

Theorem 3. For sign-changing $u \in \text{Dom}(Q_{s,\Omega}^D)$, the following relations hold:

$$Q_{s,\Omega}^N[u] > Q_{s,\Omega}^N[|u|]; \quad Q_{s,\Omega}^D[u] > Q_{s,\Omega}^D[|u|], \quad \text{if } 0 < s < 1;$$

$$Q_{s,\Omega}^D[u] < Q_{s,\Omega}^D[|u|], \quad \text{if } 1 < s < 3/2.$$

This talk is based on joint papers with Roberta Musina, see [1]–[3].

[1] R. Musina, A.I. Nazarov, On fractional Laplacians // *Comm. in PDEs*, **39** (2014), N9, 1780–1790.

[2] R. Musina, A.I. Nazarov, On fractional Laplacians-2 // *AIHP-AN*. **33** (2016), N6, 1667–1673.

[3] R. Musina, A.I. Nazarov, A note on truncations in fractional Sobolev spaces // *Bull. Math. Sci.* V. 9 (2019), N1, 1-7. DOI:10.1007/s13373-017-0107-8.

Financiado por Fundos Nacionais através da FCT – Fundação para a Ciência e a Tecnologia no âmbito do projeto UID/MAT/04561/2019

