

Disjunctive Syllogism without *Ex falso*

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Abstract
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The relation between *ex falso* and *disjunctive syllogism*, or even the justification of *ex falso* based on disjunctive syllogism, is an old topic in the History of Logic (see [2], [3], [4]). This old topic reappears in contemporary Logic since the introduction of *Minimal logic* by Johansson (see [6], [7], [8]). The disjunctive syllogism seems to be part of our general non-problematic inferential practices and superficially it doesn't seem to be related to or to depend on our acceptance of the *ex falso* rule; on the other hand, the general validity of the *ex falso* has been subjected to dispute. We know that the acceptance of the *ex falso* is a sufficient condition for the acceptance of the disjunctive syllogism and that the acceptance of the *Disjunctive-syllogism rule* implies the acceptance of the *ex falso*, as the following simple derivations in an intuitionistic natural deduction system (see [1], [5]) extended with the *Disjunctive-syllogism rule* show:

$$\frac{(A \vee B) \quad [A]^1 \quad \frac{[B]^2 \quad \neg B}{\perp} \neg\text{Elimination}}{\frac{\perp}{A} \perp_i}{A} \vee\text{Elimination } ^{1, 2}$$

$$\frac{\frac{A}{(A \vee B)} \quad \neg A}{B} \text{Disjunctive-syllogism rule}$$

The interesting question is: is the *ex falso* really a necessary condition for the acceptance of the disjunctive syllogism? The aim of the present paper is to discuss some possible ways to define systems that combines the preservation of the disjunctive syllogism with the rejection of the *ex falso*.

References

1. GENTZEN, GERHARD, *Investigations into logical deduction*, *The collected papers of Gerhard Gentzen* (Manfred Szabo, editor), North-Holland, Amsterdam, 1969, pp. 228 - 238.
2. LAERTIUS, DIOGENES, *Lives of Eminent Philosophers*, Trans. R. D. Hicks, Loeb Classical Library; London, Heinemann, 2 vols., 1925.
3. MARTIN, CHRISTOPHER, *William's Machine*, *The Journal of Philosophy*, vol.83 (1986), no.10, Eighty-Third Annual Meeting American Philosophical Association, Eastern Division, pp.564–572.
4. MATES, BENSON, *Stoic Logic*, University of California Press, Berkeley, 1953.
5. PRAWITZ, DAG, *Natural Deduction - a proof-theoretical study*, Stockholm: Almqvist & Wiksell, 1965.
6. TENNANT, NEIL, *Natural Deduction and Sequent Calculus for Intuitionistic Relevant Logic*, *The Journal of Symbolic Logic*, vol.52 (1987), no.3, pp.665–680.
7. VAN ATTEN, MARK, *On the hypothetical judgement in the history of intuitionistic logic*, *Logic, Methodology and Philosophy of Science, Proceedings of the Thirteenth International Congress* (Address of meeting), (Clark Glymour, Wang Wei and Dag Westersthul, editors), College Publications, King's College, London, 2009, pp.122–136.
8. VAN DER MOLEN, TIM, THE JOHANSSON/HEYTING LETTERS AND THE BIRTH OF MINIMAL LOGIC, *ILLC Publications, Technical Notes*. <http://www.illc.uva.nl/Research/Publications/Reports/X-2016-04.text.pdf>, Accessed: 2020-06-01