Ciências ULisboa

SEMINÁRIO

ANÁLISE E EQUAÇÕES DIFERENCIAIS

24 de Outubro | 13h30 | sala 6.2.33

Linear (and Nonlinear) Dirichlet Problems with Singular Convection/Drift Terms

Lucio Boccardo
(Sapienza Università di Roma)

Abstract:

We discuss the existence of distributional solutions for the boundary value problems (the first with a convection term, the second with a drift term)

$$(0.1)\qquad \begin{cases} -\mathrm{div}(M(x)\nabla u) = -\mathrm{div}(u\,E(x)) + f(x) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

$$(0.2)\qquad \begin{cases} -\mathrm{div}(M(x)\nabla \psi) = E(x)\,\nabla\psi + g(x) & \text{in } \Omega, \\ \psi = 0 & \text{on } \partial\Omega, \end{cases}$$

We note that, at least formally, the two above linear problems are in duality and that the differential operators may be not coercive, unless one assumes that either the norm of $|E|$ in $L^N(\Omega)$ is small, or that $\mathrm{div}(E) = 0$.

We assume that

$$E \in (L^N(\Omega))^N;\ f,\, g \in L^m(\Omega),\ m \geq 1,$$

Ω bounded, open subset of $\mathbb{R}^N$, $M : \Omega \to \mathbb{R}^{N^2}$ measurable matrix such that (for $\alpha,\, \beta \in \mathbb{R}^+$)

$$\alpha|\xi|^2 \leq M(x)\xi\,\xi, \qquad |M(x)| \leq \beta, \qquad \text{a.e. in } \Omega,\ \forall \xi \in \mathbb{R}^N$$

and we prove the same Stampacchia-Calderon-Zygmund results of the case $E = 0$.

If $E \notin (L^N(\Omega))^N$, even for nothing, as in $|E| \leq \frac{|A|}{|x|}$, $A \in \mathbb{R}$, or we add, in the boundary value problems, the zero order term "$+u$", the framework changes completely.

CMAFcIO

Financiado por Fundos Nacionais através da FCT – Fundação para a Ciência e a Tecnologia no âmbito do projeto UID/MAT/04561/2019

FCT Fundação para a Ciência e a Tecnologia