Arithmetics within the Linear Time Hierarchy

The theory $I\Delta_0$ of induction for bounded formulas in the language of arithmetic can be viewed as the union of the theories IE_n, $n > 0$ where we restrict induction to n bounded quantifier alternations, the outermost being existential. The Δ_0 formulas express exactly the linear time hierarchy sets, and so $I\Delta_0$ is often the appropriate theory to prove complexity results concerning this hierarchy. Unfortunately, the theories IE_n do not seem to correspond to natural complexity classes. On the other hand, if we add the Ω_1 axiom, $\forall x \forall y \exists z (x^{log_2 y} = z)$ to $I\Delta_0$, one gets a theory conservatively extended by Buss theory S_2 whose sub-theories S_2^i do naturally correspond to complexity classes. Namely, the Δ_i^b-predicates of S_2^i are the $P^{\Sigma_i^p}$ sets. In this talk, I will survey arithmetics for complexity classes within the linear time hierarchy and will suggest what should be the appropriate analogs to the theories S_2^i within $I\Delta_0$.