SEMINÁRIO

SISTEMAS DINÂMICOS

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A short remark to a problem on divided differences

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ABSTRACT:

Usually the functional equations for polynomials on a field $\mathbb{F}$ are written as

$$\frac{f(x)-f(y)}{x-y} = g(x+y)$$

for polynomials of second order,

$$\frac{\frac{f(x)-f(y)}{x-y} - \frac{f(y)-f(z)}{y-z}}{x-z} = g(x+y+z)$$

for polynomials of third degree,
and similarly for higher order polynomials.

At the last conference on functional equations the following observation was presented by J. Schwaiger:
Using a polynomial f of third degree, let us say

$$f(x) = a\,x^3 + b\,x^2 + c\,x + d$$

with constants a, b, c, d, we get

$$\frac{f(x)-f(y)}{x-y} = a\,(x^2+xy+y^2) + b\,(x+y) + c$$

and he added the question: What about the equation

$$\frac{f(x)-f(y)}{x-y} = g(x^2+xy+y^2) + h(x+y)$$

Are the solutions polynomials (of degree 3)?
Must the functions g, h be linear (affine)?

Partial answers (positive and negative) will be given in the talk.

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