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SEMINÁRIO DE LÓGICA MATEMÁTICA

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Modified realizability and functional interpretations: some logical and mathematical observations

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Abstract:

We review some basic and well-known facts about the possibility of extracting computational information from proofs in classical, intuitionistic and semi-intuitionistic systems. Intuitionistic reasoning is tailored to have a clear constructive content and Kleene's (numerical) realizability was a way of establishing precisely what this means for intuitionistic proofs in arithmetic. We review the variant of modified realizability due to Kreisel and see it as a form of a functional interpretation. An emphasis throughout is on the fact that there are some semi-intuitionistic principles which are amenable to computational extraction. Even Kreisel's modified realizability displays not only the constructive content of intuitionistic logic but goes a bit further since it also realizes a certain (non-intuitionistic) principle of independence of premises. This principle is a characteristic principle of modified realizability. We explain what are these characteristic principles and compare them with so-called side principles, which are also amenable to computational extraction.

Markov's principle is a characteristic principle of Gödel's functional dialectica interpretation. We review the dialectica interpretation and also analyze Markov's principle via the so-called Friedman's trick. We make a short comparison between both treatments of Markov's principle. We introduce a new interpretation (the H-interpretation: H for herbrandized) and see that LLPO (the lesser limited principle of omniscience) is one of its characteristic principles. The extraction of computational information now takes the form of bounds, not of precise witnesses. We discuss this issue and a monotonicity condition now so central in several recently defined functional interpretations. Finally, we extend the H-interpretation to the second-order arithmetical setting in such a way that WKL (weak König's lemma) turns out to be a characteristic principle.

Some open questions and some projects will be discussed during the talk.

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