

# SEMINÁRIO DE ANÁLISE E EQUAÇÕES DIFERENCIAIS

**Dia 15 de Setembro (quinta-feira), às 13h30, na sala 6.2.33**

## Problems with the operator $\nabla \times (\alpha \nabla \times \cdot)$

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Consider the problem: to find  $\mathbf{h}$  such that

$$\begin{cases} \nabla \times (\alpha(x) \nabla \times \mathbf{h}) = \mathbf{f} & \text{and} & \nabla \cdot \mathbf{h} = 0 & \text{in } \Omega, \\ + \text{ tangential or normal boundary condition,} \end{cases} \quad (\mathcal{P})$$

where  $\Omega$  is a bounded open subset of  $\mathbb{R}^d$ ,  $d \geq 2$ , with a smooth boundary,  $0 < \alpha_* \leq \alpha \leq \alpha^*$ ,  $\mathbf{f}$  belongs to  $L^p(\Omega)$  or  $W^{-1,p}(\Omega)$ ,  $1 < p < \infty$ , and the boundary condition satisfies natural assumptions.

The proof of existence, uniqueness and regularity of solution of this problem depends on the same issues for the simpler problem  $\nabla \cdot (a(x) \nabla u) = g$ , plus Dirichlet or Neumann boundary condition. We prove existence and uniqueness of strong, weak and very solutions of these problems, imposing that  $0 < a_* \leq a \leq a^*$  and suitable regularity to the given data. We obtain precise estimates for the norm of the different solutions depending on the norms of the data. In the special case of homogeneous Dirichlet boundary condition, we also study existence and uniqueness of solution in fractional Sobolev spaces.

For the case of null normal boundary condition, under different assumptions, we prove existence of strong and weak solutions of problem  $(\mathcal{P})$ . These results will be useful to study an electromagnetic induction heating problem.

Work in progress with Chérif Amrouche.

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