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SEMINÁRIO DE ANÁLISE E EQUAÇÕES DIFERENCIAIS

Dia 13 de Julho (quinta-feira), às 13h30, sala 6.2.33

Stability and attractivity for Nicholson systems with time-dependent delays

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Abstract:

In the first part of this seminar, we introduce some general concepts and results from the theory of delay differential equations (DDEs), concerning existence and uniqueness of solutions and stability for linear autonomous DDEs. As an illustration, stability properties of the scalar Nicholson's equation

$$N'(t) = -dN(t) + \beta N(t-\tau)e^{-N(t-\tau)} \quad (d, \beta, \tau > 0)$$

are presented.

Next, we concentrate our study on Nicholson-type systems with constant coefficients and multiple time-varying delays of the form

$$N_i'(t) = -d_i N_i(t) + \sum_{j=1, j\neq i}^{n} a_{ij} N_j(t) + \sum_{k=1}^{m} \beta_{ik} N_i(t - \tau_{ik}(t)) e^{-c_i N_i(t-\tau_{ik}(t))}, \ i = 1, \ldots, n, \ t \geq 0,$$

where $d_i, c_i > 0, a_{ij}, \beta_{ik} \geq 0$ with $\beta_i := \sum_{k=1}^{m} \beta_{ik} > 0$, $\tau_{ik} : [0, \infty) \to [0, \infty)$ are continuous and bounded, for $i, j = 1, \ldots, n, k = 1, \ldots, m$. Motivated by the works [1, 3], original results about the global attractivity of the positive equilibrium (when it exists) are established. First, sufficient conditions on the coefficients are given for the existence and absolute global exponential stability of a unique positive equilibrium N^*. On the other hand, upper bounds on the size of the delay functions are established, for N^* to be a global attractor of all positive solutions, improving results in [3], where a very restrictive and artificial hypothesis was imposed. In this latter case and contrarily to the situation in [3], our criterion does not require the a priori explicit knowledge of the equilibrium.

References

[1] T. Faria, G. Röst, Persistence, permanence and global stability or an n-dimensional Nicholson system, *J. Dyn. Diff. Equ.*, **26** (2014), 723–744.

[2] J. Hofbauer, An index theorem for dissipative systems. Rocky Mountain J. Math. 20 (1990), 1017–1031.

[3] R. Jia, Z. Long, M. Yang, Delay-dependent criteria on the global attractivity of Nicholson's blowflies model with patch structure, Math. Meth. Appl. Sci. **40** (2017), 4222–4232.

Remark. This is also a seminar of the Master Programme in Mathematics of the DM, FCUL, and will be presented in Portuguese.

Seminário financiado por Fundos Nacionais através da FCT – Fundação para a Ciência e a Tecnologia no âmbito do projeto UID/MAT/04561/2013

Local: FCUL – Edf. C6 - Piso 2, 6.2.33