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SEMINÁRIO DE ANÁLISE E EQUAÇÕES DIFERENCIAIS

Dia 12 de Setembro (segunda-feira), às 13h30, na sala 6.2.33

Sign-changing Solutions of the Fractional Heat Equation

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Abstract:

We consider the non-local version of the heat equation, coupled with a (possibly sign-changing) initial datum u_0:

$$\begin{cases} u_t + (-\Delta)^s u = 0 & \text{in } \mathbb{R}^N \times (0,\infty) \\ u(x,0) = u_0(x) & \text{in } \mathbb{R}^N, \end{cases} \tag{1}$$

where $s \in (1/2, 1)$ and the fractional Laplacian is defined by

$$(-\Delta)^s u(x,t) = C_{N,s}\, \mathrm{PV} \int_{\mathbb{R}^N} \frac{u(x,t) - u(y,t)}{|x-y|^{N+2s}}\, dx.$$

Non-local diffusion problems have received considerable interest recently, both due to applications in crystal dislocations and phase thansition phenomena, and to mathematically challenging features (see [3]).

Mainly by means of asymptotic estimates on the fractional heat kernel (partly proved in [2], partly new), we prove existence and uniqueness for the solution of (1) satisfying a polynomial growth condition at infinity, provided u_0 has a similar growth. Our result complements that of [1], with the growth condition replacing positivity of solutions.

Furthermore, we study geometric properties of such solutions, proving that the fractional heat flow preserves convexity of the initial datum (and something more). The path to this result involves a detailed examination of the properties of fractional Laplacians of convex functions.

Work in collaboration with Antonio Greco.

Seminário financiado por Fundos Nacionais através da FCT – Fundação para a Ciência e a Tecnologia no âmbito do projeto UID/MAT/04561/2013

Local: Departamento de Matemática, C6 - Piso 2, 6.2.33