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Monodromies, Clusters, and the WKB Approximation for q-Difference Equations

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Abstract:
The study of monodromies of differential equations has been a rich area of mathematical physics, interconnected with various fields in mathematics and physics. Recent discoveries reveal that monodromy varieties naturally possess the structure of cluster varieties, significantly enhancing our understanding of their connections to string theory and Donaldson–Thomas invariants. A key technique in these developments is the (exact) WKB approximation. In string theory, q-difference equations (qDEs) naturally appear as an "M-theory completion" of differential equations, though defining monodromy in this context remains an active research area. In this seminar, I will discuss how the WKB approximation, traditionally formulated for second-order ODEs, can be effectively generalized to second-order q-difference equations, providing a natural characterization of their monodromies. Central to this approach is the WKB Stokes diagram, known in the physics literature as the exponential network, which offers a framework for defining cluster coordinates for monodromies of qDEs.

I will illustrate this formalism through explicit examples, including the q-difference Mathieu equation. Remarkably, its monodromy around the origin—known in topological string theory as the quantum mirror map—takes the form of the Hamiltonian of a cluster integrable system in terms of these cluster coordinates.

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