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Seminário CEMAT-Ciências*

3 de Outubro - sala 6.2.33 - 16:00

Mix *-quantales and the continuous weak order

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Abstract:

The set of permutations on a finite set can be given the lattice structure known as the weak Bruhat order. This lattice structure is generalized to the set of words on a fixed alphabet Σ = { x, y, z, . . . }, where each letter has a fixed number of occurrences. These lattices are known as multinomial lattices and, when card(Σ) = 2, as lattices of lattice paths. By interpreting the letters x, y, z, . . .as axes, these words can be interpreted as discrete increasing paths on a grid of a d-dimensional cube, with d = card(Σ).

In this talk I'll explain how to extend this order to images of continuous monotone functions from the unit interval to a d-dimensional cube. The lattice so obtained is denoted L(I^d). The key tool used to realize this construction is the quantale Q∨(I) of join-continuous functions from the unit interval to itself; the construction relies on a few algebraic properties of this quantale: it is involutive (that is, cyclic, non commutiative and *-autonomous, often called a Girard quantale since it is a model of classical linear logic) and it satisfies the mix rule.

We begin developing a structural theory of the lattices L(I^d): they are self-dual, they are generated under infinite joins from their join-irreducible elements, they have no completely irreducible elements nor compact elements.
The colimit of all the d-dimensional multinomial lattices embeds into L(I^d) by taking rational coordinates.
When d = 2, L(I^d) = Q∨(I) is the Dedekind-MacNeille completion of this colimit. When d ≥ 3, every element of L(I^d) is a join of meets of elements from this colimit.

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