

# The Calculus of Natural Calculation

## – An Introduction

Gentzen [2] introduced in his seminal paper “Introduction into Logical Deduction” the calculus of Natural Deduction. His intention was to provide a formal representation of proofs as close as possible to real mathematical praxis. In our opinion, Gentzen succeeded with respect to the representation of argumentations with statements. But unfortunately, many mathematicians not only argue in their proofs (with statements), but occasionally they also calculate (with objects as numbers and sets). The usual Natural Deduction representation of such calculations (via proofs dealing with equations) seems artificial and, therefore, unnatural.

This was our motivation to propose the calculus of Natural Calculation [1], an extension of Gentzen’s Natural Deduction by proper term rules. These term rules permit the treatment of the terms of a formal language as first class members of the calculus on a par with formulae and, this way, a natural representation of calculations inside proofs.

We provide in our talk an introduction into Natural Calculation and discuss briefly its basic proof-theoretic properties up to completeness and (weak) normalisation. This introduction is complemented by some more philosophical considerations regarding proof-theoretic semantics aspects of identity; in particular, a constructive interpretation of the identity relation is suggested, which permits a better understanding of the traditional Natural Deduction rules for identity.

## References

- [1] R. Gazzari. The Calculus of Natural Calculation. *Studia Logica*, 109:1375–1411, 2021.
- [2] G. Gentzen. Investigations into logical deduction. In M. E. Szabo, editor, *The Collected Papers of Gerhard Gentzen*, Studies in Logic and the Foundations of Mathematics, pages 68–131. North-Holland, Amsterdam, 1969.